SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

MHT - CET - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

1. If sin A + cos A = 1, then sin 2A is equal to **[2]**

   a) 2 b) $\frac{1}{2}$
   c) 1 d) 0

2. The straight lines x + 2y - 9 = 0, 3x + 5y - 5 = 0 and ax + by - 1 = 0 are concurrent, if the straight line 35x - 22y + 1 = 0 passes through the point **[2]**

   a) (a, -b) b) (-a, -b)
   c) (a, b) d) (b, a)

3. Radius of the parametric equation represented by $x = 2a\left(\frac{1-t^2}{1+t^2}\right)$, $y = \frac{4at}{1+t^2}$ is **[2]**

   a) 2a b) a^2
   c) 3a d) a

4. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is **[2]**

   a) $x^2 + y^2 - 2x - 2y + 1 = 0$ b) $x^2 + y^2 + x + y - 2 = 0$
   c) $x^2 + y^2 - x - y = 0$ d) $x^2 + y^2 + 2x + 2y - 7 = 0$

5. Two events A and B have probabilities 0.25 and 0.5 respectively. The probabilities that A and B occur simultaneously is 0.15. Then the probability that A or B occurs is **[2]**

   a) 0.72 b) 0.61
   c) 0.7 d) 0.6

6. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is **[2]**

   a) 7 b) 2
   c) 0 d) 17

7. A person has 15 friends of whom 10 are relatives. In how many ways can he invite 12 guests such that 8 of them are relative? **[2]**

   a) 250 b) 175
   c) 150 d) 225

8. How many numbers can be formed from the digits 1, 2, 3, 4 when the repetition is not allowed **[2]**

   a) ${}^4P_4$ b) ${}^4P_3$
   c) ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ d) ${}^4P_1 + {}^4P_2 + {}^4P_3$

9. Let f : R $\rightarrow$ R be defined by f(x) = x^4, then **[2]**

a) f is one-one but not onto
b) f is one-one and onto
c) f is neither one-one nor onto
d) f may be one-one and onto

10. $\lim\limits_{x \to 1} \frac{x^{100}-1}{x^{50}-1} =$ **[2]**

a) -1
b) 1
c) -2
d) 2

11. If f(x) = sin x - cos x, x $\neq$ 0, is continuous at x = 0, then f(0) is equal to **[2]**

a) -1
b) 1
c) -2
d) 2

12. Which of the following is a tautology? **[2]**

a) $p \rightarrow (p \wedge q)$
b) $(p \wedge q) \leftrightarrow \sim q$
c) $q \wedge (p \rightarrow q)$
d) $\sim (p \rightarrow q) \leftrightarrow p \wedge \sim q$

13. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if **[2]**

a) $\lambda \neq$ -18
b) $\lambda \neq$ -15
c) $\lambda \neq$ -16
d) $\lambda \neq$ -17

14. If A^{-1} = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, then adj A = **[2]**

a) I
b) $2A^{-1}$
c) A^{-1}
d) A

15. The range of $\tan^{-1}$ x is **[2]**

a) $\left(\pi, \frac{\pi}{2}\right)$
b) (0, π)
c) (-π, π)
d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

16. If $\sqrt{2}$ sec θ + tan θ = 1, then the general value of θ is **[2]**

a) 2nπ - $\frac{\pi}{4}$
b) 2nπ + $\frac{\pi}{4}$
c) 2nπ $\pm$ $\frac{\pi}{4}$
d) nπ + $\frac{3\pi}{4}$

17. The perimeter of a $\triangle$ABC is 6 times the arithmetic mean of the sines of its angles. If the side a is 1, then the angle A is **[2]**

a) $\frac{\pi}{2}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) π

18. Total number of solutions of $\sin^4$ x + $\cos^4$ x = sin x cos x in [0, 2π] is equal to **[2]**

a) 8
b) 6
c) 4
d) 2

[2]

19. $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}$ dx =

a) $\frac{\pi^2}{2ab}$

b) $\frac{\pi^2}{ab}$

c) $\frac{\pi}{ab}$

d) $\frac{\pi}{2\mathrm{ab}}$

20. The value of $\int_0^{2\pi} \cos^{99} x$ dx is **[2]**

a) -1

b) 99

c) 1

d) 0

21. Let $I_1 = \int_a^{\pi-a} x f(\sin x)$ dx, $I_2 = \int_a^{\pi-a} f(\sin x)$ dx, then I_2 is equal to **[2]**

a) πI_1

b) $\frac{2}{\pi} I_1$

c) $2I_1$

d) $\frac{\pi}{2} I_1$

22. $\int_0^{\pi} \frac{dx}{1-2a\cos x+a^2}$ = **[2]**

a) $\pi(1 - a^2)$

b) $\frac{\pi}{2(1-a^2)}$

c) $2\pi(1 - a^2)$

d) $\frac{\pi}{1-a^2}$

23. If three points A, B and C have co-ordinates (1, x, 3), (3, 4, 7) and (y, -2, -5) respectively. They are collinear, then x,y = **[2]**

a) 2, 3

b) 2, -3

c) -2, 3

d) -2, -3

24. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is **[2]**

a) Zero

b) Three

c) One

d) Two

25. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors, no two of which are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then (λ being some non-zero scalar) $\vec{a} + 2\vec{b} + 6\vec{c}$ is equal to **[2]**

a) $\lambda\vec{b}$

b) $\lambda\vec{c}$

c) $\lambda\vec{a}$

d) 0

26. Line with direction ratios 1, 1, 1 is **[2]**

a) parallel to X-axis

b) parallel to Y-axis

c) equally inclined to axes

d) parallel to Z-axis

27. If the angle θ is acute, then the acute angle between $x^2(\cos\theta - \sin\theta) + 2xy\cos\theta + y^2(\cos\theta + \sin\theta) = 0$ is **[2]**

a) θ

b) $\frac{\theta}{3}$

c) 2θ

d) $\frac{\theta}{2}$

28. Equations of a line and a plane are respectively $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z+5}{1}$ and 2x - 3y + 5z = 1. Then **[2]**

a) line is parallel to the plane

b) line is equal to the plane

c) line is perpendicular to the plane

d) line lies in the plane

29. A line with positive direction cosines passes through the point P(2, -1, 2) and makes equal angles with the co-ordinate axes. The line meets the plane 2x + y + z = 9 at the point Q, then l(PQ) = **[2]**

a) $\sqrt{2}$

b) 1

c) 2

d) $\sqrt{3}$

30. The objective function z = 6x + 2y is subject to 5x + 9y $\leq$ 90, x + y $\geq$ 4, y $\leq$ 8, x, y $\geq$ 0. The minimum value of z occurs at **[2]**

a) (0, 2)

b) (0, 4)

c) (18, 0)

d) (4, 0)

31. Let f(x) = $\tan^{-1}x$. Then f'(x) + f''(x) is equal to 0, when x is equal to **[2]**

a) 1

b) 0

c) -i

d) i

32. If y = $\{f(x)\}^{\phi(x)}$, then $\frac{dy}{dx}$ is **[2]**

a) $\frac{\phi}{f}\left(\frac{df}{dx}\right)+\frac{d\phi}{dx}\log f$

b) $e^{\phi\log f}\left\{\phi\frac{f'}{f}+\phi'\log f'\right\}$

c) $e^{\phi\log f}\left\{\phi\frac{f}{f}+\phi'\log f\right\}$

d) $e^{\phi\log f}\left\{\frac{\phi}{f}\cdot\frac{df}{dx}+\log f\cdot\frac{d\phi}{dx}\right\}$

33. If y = $\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\ldots\left(1+\frac{n}{x}\right)$ and x $\neq$ 0, then $\frac{dy}{dx}$ at x = -1 is **[2]**

a) n!

b) (n - 1)!

c) $(-1)^n$ (n - 1)!

d) $(-1)^n$ n!

34. If x = sin t and y = $\sin^3$ t, then $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{2}$ is **[2]**

a) 2

b) 6

c) 4

d) 8

35. Maximum area of a rectangle whose perimeter is given as 24 metres is equal to **[2]**

a) 36 m^2

b) 49 m^2

c) 64 m^2

d) 81 m^2

36. The function f(x) = $\frac{x}{1+|x|}$ is **[2]**

a) strictly decreasing

b) neither increasing nor decreasing

c) strictly increasing

d) not differentiable at x = 0

37. The normal to the curve, x^2 + 2xy - $3y^2$ = 0 at (1, 1) **[2]**

a) does not meet the curve again

b) meets the curve again in the fourth quadrant

c) meets the curve again in the third quadrant

d) meets the curve again in the second quadrant

38. $\int e^x \tan^2(e^x)\,dx$ = **[2]**

a) tan (e^x) - e^x + c

b) e^x [tan (e^x) - 1] + c

c) tan (e^x) - x + c

d) sec (e^x) + c

39. $\int x\sqrt{1+x^2}dx =$ **[2]**

a) $3\left(1+x^2\right)^{\frac{3}{2}}+c$

b) $\sqrt{1+x^2}+c$

c) $\frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}}+\mathrm{c}$

d) $\frac{1+2x^2}{\sqrt{1+x^2}}+\mathrm{c}$

40. If $\int\frac{(2x+1)}{x^4+2x^3+x^2-1}$dx = A log $\left|\frac{x^2+x+1}{x^2+x-1}\right|$ + c, then **[2]**

a) A = $-\frac{1}{2}$

b) A = -1

c) A = $\frac{1}{2}$

d) A = 1

41. $\int\tan^{-1}\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x =$ **[2]**

a) 2 (x $\cos^{-1}$x - $\sqrt{1-x^2}$) + c

b) $\frac{1}{2}$ ($\cos^{-1}$ x - $\sqrt{1-x^2}$) + c

c) $\frac{1}{2}$ (x $\cos^{-1}$x - $\sqrt{1-x^2}$) + c

d) $\frac{1}{2}$ (x $\sin^{-1}$x - $\sqrt{1-x^2}$) + c

42. The area of the region bounded by the curve y = $\sqrt{5-x^2}$ and y = |x - 1| is **[2]**

a) $\left(\frac{3\pi}{4}-\frac{1}{2}\right)$ sq. unit

b) $\left(\frac{5\pi}{4}-\frac{1}{2}\right)$ sq. unit

c) $\left(\frac{5\pi}{4}+\frac{1}{2}\right)$ sq. unit

d) $\left(\frac{3\pi}{4}+\frac{1}{2}\right)$ sq. unit

43. The general solution of $\left(1+\mathrm{e}^{\frac{x}{y}}\right)\mathrm{d}x+\mathrm{e}^{\frac{x}{y}}\left(1-\frac{x}{y}\right)\mathrm{d}y$ = 0 is **[2]**

a) $y\mathrm{e}^{\frac{x}{y}}+y=\mathrm{c}$

b) $y\mathrm{e}^{\frac{y}{x}}+x=\mathrm{c}$

c) ye^y - x = c

d) $y\mathrm{e}^{\frac{x}{y}}+x=\mathrm{c}$

44. If y = a sin(log x) + b cos(log x), then the differential equation without the parameter a and b is **[2]**

a) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$

b) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$

c) $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$

d) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2y=0$

45. If xdy = y(dx + ydy), y > 0 and y(1) = 1, then y(-3) is equal to **[2]**

a) -1

b) 5

c) 3

d) 1

46. A pair of dice is thrown and X denotes the sum of the numbers on uppermost faces. Then the expected value of X is **[2]**

a) 6

b) 12

c) 8

d) 7

47. The p.d.f. of a random variable X is f(x) = $\begin{cases}\frac{1}{5}; & 0\le x\le 5\\ 0; & \text{otherwise}\end{cases}$ then the value of P(1 < X < 3) is **[2]**

a) $\frac{3}{5}$

b) $\frac{4}{5}$

c) $\frac{2}{5}$

d) $\frac{1}{5}$

48. For the probability distribution given by **[2]**

$X = x_i$	0	1	2
P_i	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

the standard deviation (σ) is

a) $\sqrt{\frac{1}{4}}$

b) $\sqrt{\frac{5}{36}}$

c) $\sqrt{\frac{1}{3}}$

d) $\frac{1}{3}\sqrt{\frac{5}{2}}$

49. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than or equal to 1 is **[2]**

a) $\frac{2}{3}$

b) $\frac{7}{8}$

c) $\frac{4}{5}$

d) $\frac{15}{16}$

50. If getting a head on a coin when it is tossed is considered as success, then the probability of having more number of failures when ten fair coins are tossed simultaneously, is **[2]**

a) $\frac{105}{2^8}$

b) $\frac{73}{2^7}$

c) $\frac{638}{2^{10}}$

d) $\frac{193}{2^9}$