Solution

GEOMERTY

Class 10 - Mathematics - II

1. (i) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

   i. **(c)** $(3,4,5)$

   Explanation:
   $5^2=25$
   $3^2+4^2=9+16=25$
   $\therefore 5^2=3^2+4^2$

   ii. **(c)** 1

   Explanation:
   $2\tan 45^{\circ}-2\sin 30^{\circ}$
   $=2(1)-2\left(\frac{1}{2}\right)$
   $=2-1=1$

   iii. **(d)** $\sqrt{3}$

   Explanation:
   Angle made with the positive direction of X-axis $(\theta)=60^{\circ}$
   Slope of the line $(m)=\tan\theta$
   $\therefore m=\tan 60^{\circ}=\sqrt{3}$
   $\therefore$ The slope of the line is $\sqrt{3}$.

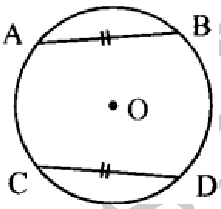
   iv. **(d)** 8.8 cm

   Explanation:
   Distance between centres $=5.5+3.3$
   = 8.8

   (ii) Solve the following sub-questions :

   i. $\frac{A(\triangle ABC)}{A(\triangle PQR)}=\frac{AB^2}{PQ^2}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$ ...[Theorem of areas of similar triangles]

   ii.



   In a circle with centre O,
   chord $AB\cong$ chord CD
   $\therefore m(\operatorname{arc} AB)\cong m(\operatorname{arc} CD)$ ...[Corresponding arcs of congruent chords of a circle are congruent]
   $\therefore m(\text{arcCD})=120^{\circ}$

   iii. Diagonal of a square $=\sqrt{2}\times$ side
   $=\sqrt{2}\times 10$
   $=10\sqrt{2}\ cm$

   iv. Let $A\left(x_1,y_1\right)=A(4,-3)$
   $B\left(x_2,y_2\right)=B(7,5)$
   $C\left(x_3,y_3\right)=C(-2,1)$
   $\therefore$ By centroid formula,
   $y=\frac{y_1+y_2+y_3}{3}=\frac{-3+5+1}{3}=\frac{3}{3}=1$
   $\therefore$ The Y-co-ordinate of the centroid of the given triangle is 1.

2. (i) Complete any two activities and rewrite it :

   i. $m(\operatorname{arc} PQR)=180^{\circ}\ldots$ [measure of semicircle]
   $\therefore m(\text{arc PXR})=\boxed{180^{\circ}}$

$\therefore \angle PQR = \frac{1}{2}\, m(\text{arc}\ \boxed{\text{PXR}})$ $\boxed{\text{... Inscribed angle theorem}}$
$= \frac{1}{2} \times 180^\circ$
$\therefore \angle PQR = \boxed{90^\circ}$

ii. $\cos\theta = \frac{5}{13}$ ...[Given]
We know that,
$\sin^2\theta + \cos^2\theta = 1$
$\therefore \sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$
$\therefore \sin^2\theta + \frac{25}{169} = 1$
$\therefore \sin^2\theta = 1 - \frac{25}{169}$
$\therefore \sin^2\theta = \frac{169-25}{169}$
$\therefore \sin^2\theta = \frac{144}{169}$
$\therefore \sin\theta = \frac{12}{13}$ ... [Taking square root on both sides]

iii. Given: Radius (r) = 7cm, Length of arc (l) = 20 cm
To find: Area of the sector
Area of sector $= \frac{l \times r}{2}$
$= \frac{20\times 7}{2}$
$= 10 \times 7 = 70\ cm^2$
$\therefore$ The area of the sector is $70\ cm^2$.

(ii) Solve any four of the following sub-questions :

i. Chord AB and chord CD intersect each other at point E ...(given)
$\therefore CE \times ED = AE \times EB$
... $\boxed{\text{Theorem of internal division of chords}}$
$\therefore \boxed{12} \times ED = 15 \times 6$
$\therefore ED = \frac{\boxed{90}}{12}$
$\therefore ED = \boxed{7.5}$

ii. $\triangle ABC$ is $\boxed{right\ angled}$ triangle.
$\therefore$ By Pythagoras theorem,
$AB^2 + BC^2 = AC^2$
$\therefore 5^2 + BC^2 = 13^2$
$\therefore 25 + BC^2 = \boxed{169}$
$\therefore BC^2 = 169 - 25 = \boxed{144}$
$\therefore BC = \boxed{12 \text{ units}}$

iii. i. $\angle L = \frac{1}{2}\, m(\text{arc}\, MN)$ ...[By inscribed angle theorem]
$\therefore \boxed{35} = \frac{1}{2}\, m(\text{arc MN})$
$\therefore 2 \times 35 = m(\text{arc}\, MN)$
$\therefore m(\text{arc}\, MN) = \boxed{70^\circ}$

ii. $m(\text{arc}\, MLN) = \boxed{360^\circ} - m(\text{arc}\, MN)$ ...[Definition of measure of arc]
$= 360^\circ - 70^\circ$
$\therefore m(\text{arc}\, MLN) = \boxed{290^\circ}$

iv. Let $A(x_1, y_1) = A(2, 5)$ and $B(x_2, y_2) = B(4, -1)$
Here, $x_1 = 2, x_2 = 4, y_1 = 5, y_2 = -1$
Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-1-5}{4-2} = \frac{-6}{2} = -3$
$\therefore$ The slope of line AB is -3.

v. Area of sector $= \frac{l \times r}{2}$
Here, $l = 2.2\ cm$ and $r = 3.5\ cm$
$\therefore$ Area of sector $= \frac{2.2\times 3.5}{2}$
$= 1.1 \times 3.5$

$= 3.85\ cm^2$

$\therefore$ The area of the sector is $3.85\ cm^2$.

3. (i) Complete any one activity of the following and rewrite it :

i. $2\text{AX} = 3\text{BX}$ ...[Given]

$\therefore \frac{AX}{BX} = \frac{3}{2}$

$\therefore \frac{AX+BX}{BX} = \frac{3+2}{2}$ ...[By componendo]

$\therefore \frac{BA}{BX} = \frac{5}{2} \ldots$ (i) $[A - X - B]$

In $\triangle BCA$ and $\triangle BYX$,

$\left.\begin{array}{l}\angle BCA \cong \angle BYX \\ \angle BAC \cong \angle BXY\end{array}\right\} \ldots$ [Corresponding angles]

$\therefore \triangle BCA \sim \triangle BYX \ldots$ [By AA test of similarity]

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ ...[corresponding sides of similar triangles]

$\therefore \frac{5}{2} = \frac{AC}{9}$ ... [From (i)]

$\therefore AC = \frac{9\times5}{2}$

$\therefore AC = 22.5$ units

ii. In $\triangle CAE$ and $\triangle BDE$.

$\angle AEC \cong \angle DEB \ldots.$ Vertically opposite angles

$\angle CAE \cong \angle BDE$ ...[angles inscribed in the same arc]

$\therefore \triangle CAE \sim \triangle BDE \ldots$ AA test of similarity

$\therefore \frac{\boxed{AE}}{DE} = \frac{CE}{\boxed{BE}}$ ... [corresponding sides of similar triangles]

$\therefore AE \times EB = CE \times ED.$

(ii) Solve any two of the following sub-questions :

i. Distance between two points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

By distance formula,

$AB = \sqrt{[2-(-2)]^2 + (2-2)^2}$

$= \sqrt{(2+2)^2 + (0)^2}$

$= \sqrt{(4)^2} = 4$

$BC = \sqrt{(2-2)^2 + (7-2)^2}$

$= \sqrt{(0)^2 + (5)^2}$

$= \sqrt{(5)^2} = 5$

$AC = \sqrt{[2-(-2)]^2 + (7-2)^2}$

$= \sqrt{(2+2)^2 + (5)^2} = \sqrt{(4)^2 + (5)^2}$

$= \sqrt{16+25} = \sqrt{41}$

Now, $AC^2 = (\sqrt{41})^2 = 41$

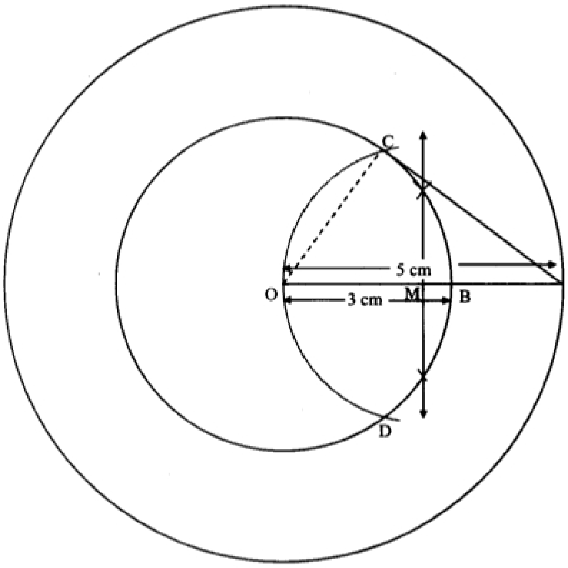
Consider, $AB^2 + BC^2 = 4^2 + 5^2$

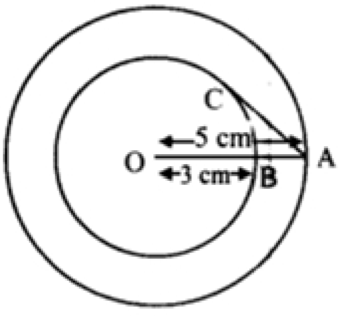
$= 16 + 25 = 41$

$\therefore AC^2 = AB^2 + BC^2$

$\therefore \triangle ABC$ is a right angled triangle. ...[Converse of Pythagoras theorem]

$\therefore$ Points A, B and C are the vertices of a right angled triangle.

ii. 



Rough figure

Steps of construction:

i. With centre O, draw concentric circles with radii 3 cm and 5 cm.
ii. Take point A such that $OA = 5\ cm$.
iii. Draw the perpendicular bisector of seg OA. It intersects OA in point M.
iv. With M as centre and radius equal to AM, draw an arc intersecting the circle in points C and D.
v. Draw ray AC.
Ray AC is the required tangent.

Length of the tangent segment is $4\ cm$**.**

By Pythagoras theorem:

Tangent $CA \perp$ radius OC ...[Tangent theorem]

$\therefore$ In $\triangle AOC, \angle C = 90^\circ$

$OA = 5\ cm$ ...[Radius of the bigger circle]

$OC = 3\ cm$ ...[Radius of the smaller circle]

$\therefore$ By Pythagoras Theorem, we get

$OA^2 = OC^2 + AC^2$

$\therefore (5)^2 = (3)^2 + AC^2$

$\therefore 25 = 9 + AC^2$

$\therefore AC^2 = 25 - 9$

$\therefore AC^2 = 16$

$\therefore AC = 4\ cm$

$\therefore$ Length of the tangent segment is $4\ cm$.

iii. In $\triangle PSQ, \angle PSQ = 90^\circ$

$\therefore PS^2 + QS^2 = PQ^2 \ldots$ [Pythagoras theorem]

$\therefore PS^2 = PQ^2 - \boxed{QS^2}$ ...(i)

Similarly,

In $\triangle PSR, \angle PSR = 90^\circ$

$\therefore PS^2 + \boxed{RS^2} = PR^2$ ...[Pythagoras theorem]

$\therefore PS^2 = PR^2 - \boxed{RS^2}$ ...(ii)

$\therefore PQ^2 - \boxed{QS^2} = \boxed{PR^2} - RS^2 \ldots$ [From (i) and (ii)]

$\therefore PQ^2 + \boxed{RS^2} = PR^2 + QS^2$

4. Solve any two of the following sub-questions :

(i) Let AB be the tower of height 48 m.

Let C and D be the position of two cars at angles of depression 60° and 30° respectively.

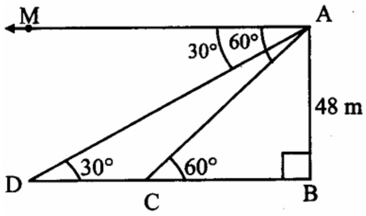
AM is the horizontal line and $\angle MAD$ and $\angle MAC$ are angles of depression of measures 30° and 60° respectively.

$\angle MAD$ and $\angle ADB$, $\angle MAC$ and $\angle ACB$ are pairs of alternate angles.

$\therefore m\angle ADB = 30^\circ$ and $m\angle ACB = 60^\circ$

To find: Distance between C and D.

i.e., $l(\text{seg } CD)$



In $\triangle ADB, \angle B = 90^\circ$

$\therefore \tan 30^\circ = \frac{AB}{DB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{48}{DB}$

$\therefore DB = 48\sqrt{3}\ cm$

In $\triangle ACB, \angle B = 90^\circ$

$\therefore \tan 60^\circ = \frac{AB}{CB}$

$\therefore \sqrt{3} = \frac{48}{CB}$

$\therefore CB = \frac{48}{\sqrt{3}}$

$\therefore CB = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ...[Multiply numerator and denominator by $\sqrt{3}$]

$\therefore CB = \frac{48\sqrt{3}}{3}$

$\therefore CB = 16\sqrt{3}\ cm$

$\therefore DC = DB - CB$

$= 48\sqrt{3} - 16\sqrt{3}$

$= 32\sqrt{3}$

$= 32 \times 1.73 \ldots [\sqrt{3} = 1.73]$

$= 55.36\ cm$

$\therefore$ The distance between the two cars is $55.36\ cm$.

(ii) Area of square $ABCD = (\text{side})^2$

$= (50)^2$

$= 2500\ m^2$

Radius of sector $A - SP = \frac{1}{2} \times 50 = 25\ m$

$\theta = 90^\circ$ ...[Angle of a square]

Area of sector $A - SP = \frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times \frac{22}{7} \times (25)^2$

$= \left(\frac{1}{4} \times \frac{13750}{7}\right) m^2$

A(shaded region)

$=$ Area of square $ABCD$ $-4($ Area of sector $A - SP)$

$= 2500 - 4\left(\frac{1}{4} \times \frac{13750}{7}\right)$

$= 2500 - \frac{13750}{7}$

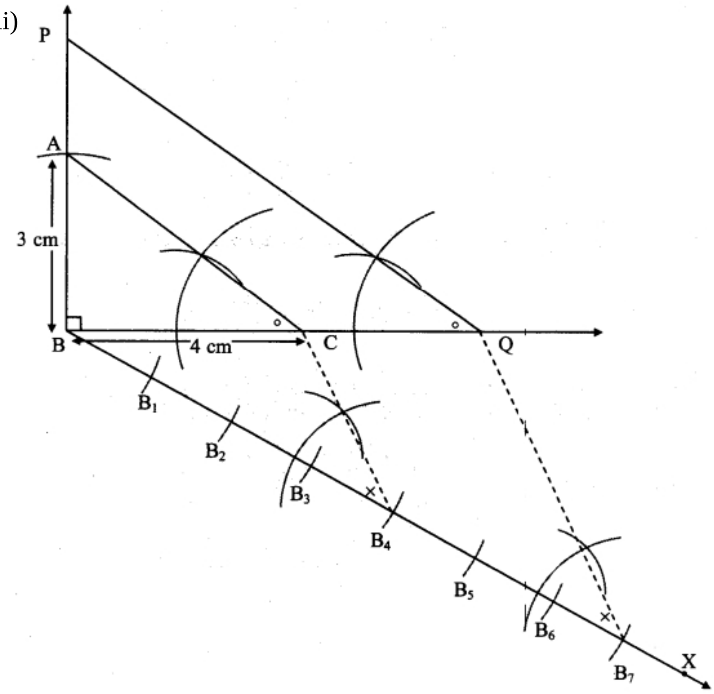
$= \frac{17500 - 13750}{7}$

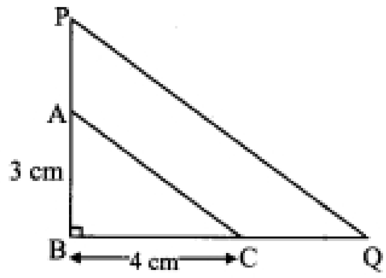
$= \frac{3750}{7}$

$\approx 535.71\ m^2$

Area of the shaded region is $535.71\ m^2$.

(iii)





Rough Figure

Steps of construction:

i. Draw segment BC of length 4 cm.
ii. Take $\angle B$ as 90° and draw an arc of 3 cm on it. Name the point as A.
iii. Join segment AC to obtain $\triangle ABC$.
iv. Draw ray BX such that $\angle CBX$ is an acute angle.
v. Locate points $B_1, B_2, B_3, B_4, B_5, B_6, B_7$ on ray BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.
vi. Join point C and B_4.
vii. Through point B_7 draw a line parallel to segment CB_4 which intersects segment BC at point Q.
viii. Draw a line parallel to AC through Q to intersect line AB at point P.
$\triangle PBQ$ is the required triangle similar to $\triangle ABC$.

5. Solve any one of the following sub-questions :

(i) In $\triangle XDE, PQ \| DE$ ...[Given]

$\therefore \frac{XP}{\boxed{PD}} = \frac{\boxed{XQ}}{QE}$ ...(i) [Basic proportionality theorem]

In $\triangle XEF, QR \| EF$ ...[Given]

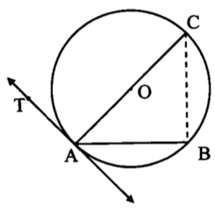
$\therefore \frac{XQ}{QE} = \frac{\boxed{XR}}{RF}$ ... (ii) [Basic proportionality theorem]

$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ From (i) and (ii)]

$\therefore$ seg PR $\|$ seg DF ...[Converse of basic proportionality theorem]

(ii)

i.



ii. AOC is a diameter of the circle with centre O . ...[Given]
$\therefore$ seg OA is a radius.
$\therefore$ seg OA $\perp$ line TA ...[Tangent theorem]
$\therefore \angle OAT = 90^\circ$ ...(i) [A-O-C]
As AC is a diameter, we get
$\therefore \angle ABC = 90^\circ$ ... (ii)[Angle inscribed in a semicircle]

iii. From equations (i) and (ii), we get
$\angle CAT \cong \angle ABC$