Solution

GEOMERTY

Class 10 - Mathematics - II

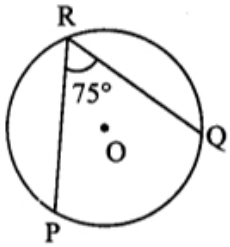
1. (i) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

   i. **(c)** Right angled triangle
   Explanation:
   Right angled triangle

   ii. **(b)** $\sec^2\theta$
   Explanation:
   $\sec^2\theta$

   iii. **(d)** $\sqrt{3}$
   Explanation:
   Angle made with the positive direction of X-axis $(\theta) = 60^\circ$
   Slope of the line $(m) = \tan\theta$
   $\therefore m = \tan 60^\circ = \sqrt{3}$
   $\therefore$ The slope of the line is $\sqrt{3}$.

   iv. **(d)** 210°
   Explanation:
   $m(\text{arc PRQ})$
   $= 360^\circ - 2(\ m\angle PRQ)$
   $= 360^\circ - 2 \times 75^\circ$
   $= 360^\circ - 150^\circ$
   $= 210^\circ$



(ii) Solve the following sub-questions :

i. $\triangle ABC$ and $\triangle DCB$ have same base BC.
$\therefore \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{AB}{DC}$ ... [Triangles having equal base]
$= \frac{6}{8}$
$\therefore \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{3}{4}$

ii. As opposite angles of a cyclic quadrilateral are supplementary.
$\therefore \angle B + \angle D = 180^\circ$
$\therefore 110^\circ + \angle D = 180^\circ$
$\therefore \angle D = 180^\circ - 110^\circ$
$\therefore \angle D = 70^\circ$

iii. In $\triangle ABC, \angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ$ ...[Given]
$\therefore AB = \frac{1}{\sqrt{2}} AC \ldots [By 45^\circ - 45^\circ - 90^\circ$ Theorem]
$\therefore AB = \frac{1}{\sqrt{2}} \times 9\sqrt{2}$
$\therefore AB = 9$ units

iv. Here $O(0, 0)$ and $P(3, 4)$
$\therefore x_1 = 0, y_1 = 0, x_2 = 3, y_2 = 4$
By distance formula,
$d(O, P) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

SA-ISH SCIENCE ACADEMY

$= \sqrt{(3-0)^2 + (4-0)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$\therefore d(O, P) = 5$ units.

$\therefore$ The distance between the points O and P is 5 units.

2. (i) Complete any two activities and rewrite it :

i. i. $m(\text{arc } EF) = m\angle ECF$ ...[Definition of measure of minor arc]

$\therefore m(\text{arc } EF) = 70^\circ$

$m(\text{arc } DE) + m(\text{arc } DGF) + m(\text{arc } EF) = 360^\circ$ ...[Measure of a circle is 360°]

$\therefore m(\text{arc } DE) = 360^\circ - m(\text{arc } DGF) - m(\text{arc } EF)$

$= 360^\circ - 200^\circ - 70^\circ$

$\therefore m(\text{arc } DE) = 90^\circ$

ii. $m(\text{arc } DEF) = m(\text{arc } DE) + m(\text{arc } EF)$ ...[Arc addition property]

$= 90^\circ + 70^\circ$

$\therefore m(\text{arc DEF}) = 160^\circ$

ii. L.H.S. $= \cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\boxed{\cos^2\theta} + \boxed{\sin^2\theta}}{\sin\theta \times \cos\theta}$

$= \frac{1}{\sin\theta \times \cos\theta}$ ...$\boxed{[\because \sin^2\theta + \cos^2\theta = 1]}$

$= \frac{1}{\sin\theta} \times \frac{1}{\boxed{\cos\theta}}$

$= \text{cosec}\,\theta \times \sec\theta$

L.H.S. = R.H.S.

$\therefore \cot\theta + \tan\theta = \text{cosec}\,\theta \times \sec\theta$

iii. Area of square $= (\boxed{\text{side}})^2$ . . . [Formula]

$= 14^2$

$= \boxed{196}\ cm^2$

Area of circle $= \boxed{\pi r^2}$ . . . [Formula]

$= \frac{22}{7} \times 7 \times 7$

$= 154\ cm^2$

Area of shaded portion

= Area of square − Area of circle

$= 196 - 154$

$= \boxed{42}\ cm^2$

(ii) Solve any four of the following sub-questions :

i. i. $\angle L = \frac{1}{2}\ m(\text{arc } MN)$ ...[By inscribed angle theorem]

$\therefore \boxed{35} = \frac{1}{2}\ m(\text{arc MN})$

$\therefore 2 \times 35 = m(\text{arc } MN)$

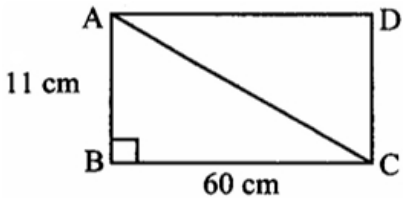
$\therefore m(\text{arc } MN) = \boxed{70^\circ}$

ii. $m(\text{arc } MLN) = \boxed{360^\circ} - m(\text{arc } MN)$ ...[Definition of measure of arc]

$= 360^\circ - 70^\circ$

$\therefore m(\text{arc } MLN) = \boxed{290^\circ}$

ii.



Let $\square ABCD$ be the given rectangle.

$AB = 11\ cm, BC = 60\ cm$

In $\triangle ABC, \angle B = 90^\circ$ . . . [Angle of a rectangle]

$\therefore AC^2 = AB^2 + BC^2$ . . . [Pythagoras theorem]

SATISH SCIENCE ACADEMY

$= 11^2 + 60^2$

$= 121 + 3600$

$= 3721$

$\therefore AC = \sqrt{3721}$ ...[Taking square root of both sides]

$= 61\ cm$

$\therefore$ The length of the diagonal of the rectangle is 61 cm.

iii. Ray PQ is a tangent to the circle at point Q and seg PS is the secant. ...[Given]

$\therefore PR \times PS = PQ^2$ ...[Tangent secant segments theorem]

$\therefore 8 \times PS = 12^2$

$\therefore 8 \times PS = 144$

$\therefore PS = \frac{144}{8}$

$\therefore$ PS = 18 units

iv. $A(x_1, y_1) = A(2, 3)$ and $B(x_2, y_2) = B(4, 7)$

Here, $x_1 = 2, x_2 = 4, y_1 = 3, y_2 = 7$

Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2} = \frac{4}{2} = 2$

$\therefore$ The slope of line AB is 2.

v. Area of square $ABCD$ = $\boxed{(side)^2}$

$= 14^2 = 196\ cm^2$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 7^2$

$= \boxed{154} cm^2$

Area of shaded portion

= Area of square ABCD - Area of circle

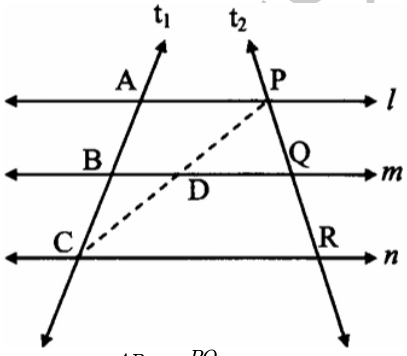
$= 196 - \boxed{154}$

$= \boxed{42}\ cm^2$

3. (i) Complete any one activity of the following and rewrite it :

i. Given: line $l \parallel$ line $m \parallel$ line n

Transversals t_1 and t_2 intersect the parallel lines at points A, B, C and P, Q, R respectively.



To prove: $\frac{AB}{BC} = \frac{PQ}{QR}$

Construction: Draw seg PC which intersects line m at point D.

Proof: In $\triangle ACP$, seg $BD \parallel$ seg AP ...[Given]

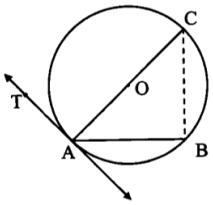
$\therefore \frac{AB}{BC} = \frac{PD}{DC}$ ... (i)[Basic proportionality theorem]

In $\triangle RPC$, seg $DQ \parallel$ seg CR ...[Given]

$\therefore \frac{PQ}{QR} = \frac{PD}{DC}$ ... (ii)[Basic proportionality theorem]

$\therefore \frac{AB}{BC} = \frac{PQ}{QR}$ ... [From (i) and (ii)]

ii. i.



SH SCIENCE ACADEMY

ii. AOC is a diameter of the circle with centre O . ...[Given]

$\therefore$ seg OA is a radius.

$\therefore$ seg OA $\perp$ line TA ...[Tangent theorem]

$\therefore \angle OAT = 90^\circ$ ...(i) [A-O-C]

As AC is a diameter, we get

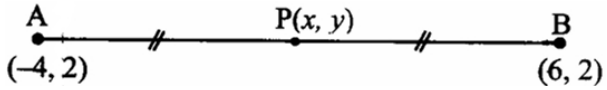
$\therefore \angle ABC = 90^\circ$ ... (ii)[Angle inscribed in a semicircle]

iii. From equations (i) and (ii), we get

$\angle CAT \cong \angle ABC$

(ii) Solve any two of the following sub-questions :

i.



Suppose, $(-4, 2) = (x_1, y_1)$ and $(6, 2) = (x_2, y_2)$ and co-ordinates of P are (x, y)

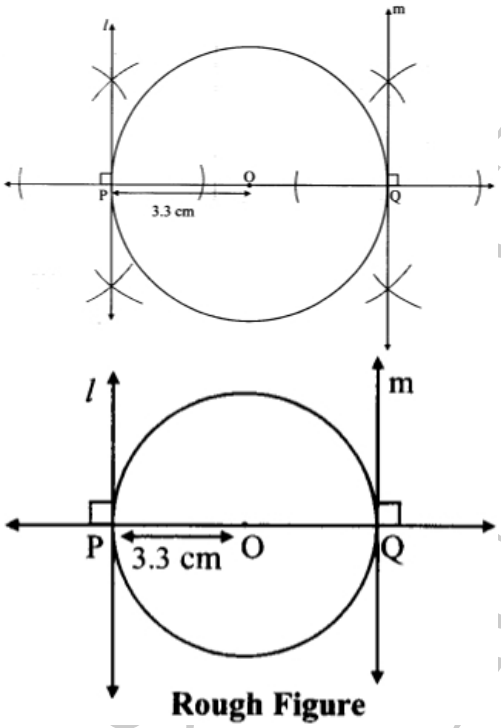
$\therefore$ According to midpoint theorem,

$x = \frac{x_1+x_2}{2} = \frac{\boxed{-4}+6}{2} = \frac{\boxed{2}}{2} = \boxed{1}$

$y = \frac{y_1+y_2}{2} = \frac{2+\boxed{2}}{2} = \frac{4}{2} = \boxed{2}$

$\therefore$ Co-ordinates of midpoint P are $\boxed{(1, 2)}$

ii.



Rough Figure

Steps of construction:

i. With centre O, draw a circle of radius 3.3 cm.

ii. Draw a chord PQ of length 6.6 cm.

iii. Draw rays OP and OQ.

iv. Draw line $l \perp$ ray OP at point P.

v. Draw line $m \perp$ ray OQ at point Q.

Lines l and m are the required tangents to the circle.

iii. $\left(a^2 + b^2\right)^2 = a^4 + 2a^2b^2 + b^4 \ldots$ (i)

$\left(a^2 - b^2\right)^2 = a^4 - 2a^2b^2 + b^4 \ldots$ (ii)

$(2ab)^2 = 4a^2b^2 \ldots$ (iii)

Now,

$\left(a^4 + 2a^2b^2 + b^4\right) = \left(a^4 - 2a^2b^2 + b^4\right) + 4a^2b^2$

$\therefore \left(a^2 + b^2\right)^2 = \left(a^2 - b^2\right)^2 + (2ab)^2$ ...[From (i), (ii) and (iii)]

$\therefore \left[\left(a^2 + b^2\right), \left(a^2 - b^2\right), (2ab)\right]$ is a Pythagorean triplet.

The triangle with sides $\left(a^2 + b^2\right), \left(a^2 - b^2\right)$ and (2ab) is a right angled triangle.

4. Solve any two of the following sub-questions :

SCIENCE ACADEMY

(i) Let AB be the tower of height 48 m.

Let C and D be the position of two cars at angles of depression 60° and 30° respectively.

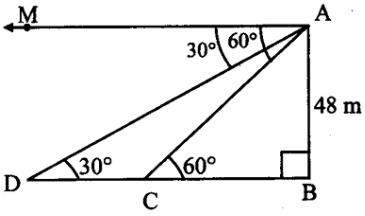
AM is the horizontal line and $\angle MAD$ and $\angle MAC$ are angles of depression of measures 30° and 60° respectively.

$\angle MAD$ and $\angle ADB$, $\angle MAC$ and $\angle ACB$ are pairs of alternate angles.

$\therefore m\angle ADB = 30^\circ$ and $m\angle ACB = 60^\circ$

To find: Distance between C and D.

i.e., $l(\text{seg}\, CD)$



In $\triangle ADB, \angle B = 90^\circ$

$\therefore \tan 30^\circ = \frac{AB}{DB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{48}{DB}$

$\therefore DB = 48\sqrt{3}\ cm$

In $\triangle ACB, \angle B = 90^\circ$

$\therefore \tan 60^\circ = \frac{AB}{CB}$

$\therefore \sqrt{3} = \frac{48}{CB}$

$\therefore CB = \frac{48}{\sqrt{3}}$

$\therefore CB = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ...[Multiply numerator and denominator by $\sqrt{3}$]

$\therefore CB = \frac{48\sqrt{3}}{3}$

$\therefore CB = 16\sqrt{3}\ cm$

$\therefore DC = DB - CB$

$= 48\sqrt{3} - 16\sqrt{3}$

$= 32\sqrt{3}$

$= 32 \times 1.73 \ldots [\sqrt{3} = 1.73]$

$= 55.36\ cm$

$\therefore$ The distance between the two cars is $55.36\ cm$.

(ii) Radius of the top of the frustum $(r_1) = 28\ cm$

Radius of the bottom of the frustum $(r_2) = 21\ cm$

Capacity of bucket = Volume of frustum

$\therefore 28.490$ litres $= \frac{1}{3}\pi\, h\left(r_1^2 + r_2^2 + r_1 \times r_2\right)$

$\therefore 28.490 \times 1000 = \frac{1}{3} \times \frac{22}{7} \times h\left(28^2 + 21^2 + 28 \times 21\right) \ldots \left[\because 1 \text{ litre } = 1000\ cm^3\right]$

$\therefore 28490 = \frac{22h}{3\times 7}(784 + 441 + 588)$

$\therefore 28490 = \frac{22h}{21}(1813)$

$\therefore h = \frac{28490\times 21}{22\times 1813}$

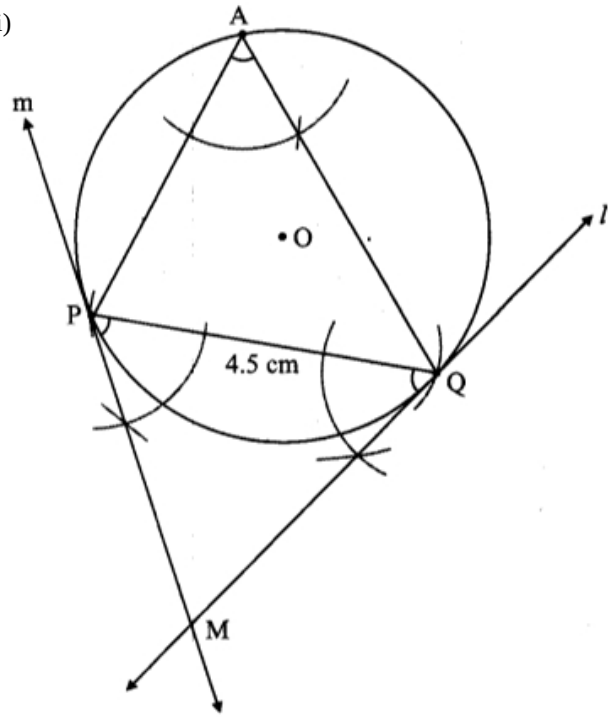
$= \frac{1295\times 3}{259}$

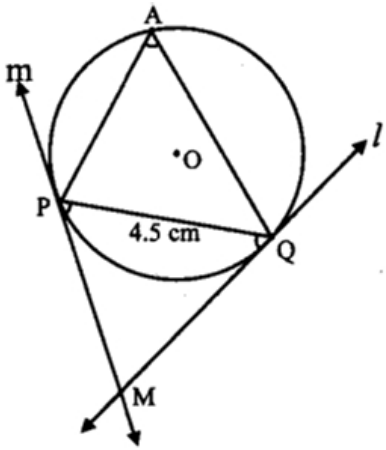
$= 5 \times 3 = 15\ cm$

$\therefore$ Height of the bucket is 15 cm.

SATISH SCIENCE ACADEMY

(iii)





Rough Figure

Steps of construction:

i. Draw a circle of radius 2.7 cm.
ii. Draw a chord PQ of length 4.5 cm.
iii. Take a point A on the major arc, other than P and Q.
iv. Join PA and QA.
v. Using P and Q as vertices and chord PQ as one side, draw $\angle MPQ$ and $\angle MQP$ equal to $\angle PAQ$.
vi. Lines containing the rays PM and QM are the tangents to the circle at P and Q respectively.

5. Solve any one of the following sub-questions :

(i) In $\triangle ABC$, ray BD bisects $\angle B$

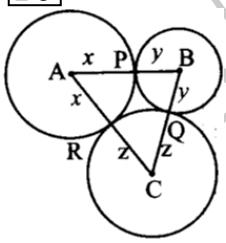
$\therefore \frac{\boxed{AB}}{BC} = \frac{AD}{DC}$ ...(i) Angle bisector theorem

In $\triangle ABC, DE \| BC$

$\therefore \frac{\boxed{AE}}{EB} = \frac{AD}{DC}$ ...(ii) Basic proportionality theorem

$\frac{AB}{\boxed{BC}} = \frac{\boxed{AE}}{EB}$ ...[From (i) and (ii)]

(ii)



$$\text{Let } \begin{matrix} AP & = AR = x \\ BP & = BQ = y \\ CQ & = CR = z \end{matrix} \Big\} \text{ [Radii of the same circle]}$$

AP + BP = AB ...[A - P - B]

$\therefore x + y = 3 \ldots$ (i)

$BQ + CQ = BC \ldots [B - Q - C]$

$\therefore y + z = 3 \ldots$ (ii)

$AR + CR = AC \ldots [(A - R - C]$

$\therefore x + z = 4 \ldots$ (iii)

Adding equations (i), (ii) and (iii), we get

$x + y + y + z + x + z = 3 + 3 + 4$

$\therefore 2x + 2y + 2z = 10$

$\therefore 2(x + y + z) = 10$

JAYESH SCIENCE ACADEMY

$\therefore x + y + z = 5$ ...(iv)

Substituting equation (i) in equation (iv), we get

$3 + z = 5$

$\therefore z = 5 - 3$

$\therefore z = 2\ cm$

Substituting equation (ii) in equation (iv), we get

$x + 3 = 5$

$\therefore x = 5 - 3$

$\therefore x = 2\ cm$

Substituting equation (iii) in equation (iv), we get

$y + 4 = 5$

$\therefore y = 5 - 4$

$\therefore y = 1\ cm$

$\therefore$ The radii of circles with centres A, B, C are $2\ cm, 1\ cm$ and 2 cm respectively.

SATISH SCIENCE ACADEMY