Solution

GEOMERTY

Class 10 - Mathematics - II

1. (i) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

   i. **(d)** 13

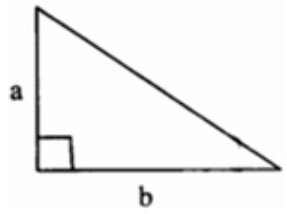
   Explanation:

   According to the given condition, we get

   $a^2 + b^2 = 169$

   $\therefore$ Length of hypotenuse

   $= \sqrt{169} = 13$ units

   ii. **(b)** 60°

   Explanation:

   $\tan\theta = \sqrt{3}$

   Also, $\tan 60^\circ = \sqrt{3}$

   $\therefore \tan\theta = \tan 60^\circ$

   $\Rightarrow \theta = 60^\circ$

   iii. **(b)** 5

   Explanation:

   Distance of $(-3, 4)$ from origin

   $= \sqrt{(-3)^2 + (4)^2}$

   $= \sqrt{9 + 16}$

   $= \sqrt{25} = 5 \ldots$ [As distance can't be negative]

   iv. **(d)** 8.8 cm

   Explanation:

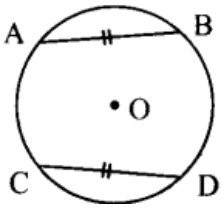
   Distance between centres $= 5.5 + 3.3$

   = 8.8

(ii) Solve the following sub-questions :

i. $\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ ...[Theorem of areas of similar triangles]

ii.



In a circle with centre O,

chord $AB \cong$ chord CD

$\therefore m(\text{arc}\, AB) \cong m(\text{arc}\, CD)$ ...[Corresponding arcs of congruent chords of a circle are congruent]

$\therefore m(\text{arcCD}) = 120^\circ$

iii. Diagonal of a square $= \sqrt{2} \times$ side

$= \sqrt{2} \times 10$

$= 10\sqrt{2}\ cm$

iv. Let $A(x_1, y_1) = A(22, 20)$

$B(x_2, y_2) = B(0, 16)$

Let the co-ordinates of the midpoint be $P(x, y)$.

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$\therefore$ By midpoint formula,

$x = \frac{x_1+x_2}{2} = \frac{22+0}{2} = 11$

$y = \frac{y_1+y_2}{2} = \frac{20+16}{2} = \frac{36}{2} = 18$

$\therefore$ The co-ordinates of the midpoint of the segment joining $(22, 20)$ and $(0, 16)$ are $(11, 18)$.

2. (i) Complete any two activities and rewrite it :

i. $m\angle PQR = \frac{1}{2}\, m(\text{arc}\, PXR)$ ...(i) [Inscribed angle theorem]

$m\angle \boxed{PSR} = \frac{1}{2}\, m(\text{arc PXR})$ ...(ii) [Inscribed angle theorem]

$\therefore m\angle \boxed{PQR} = m\angle PSR$ ... [From (i) and (ii)]

$\therefore \angle PQR \cong \angle PSR$ ...[Angles equal in measure are congruent]

ii. $\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\left(\frac{3}{5}\right)^2}$

$= \sqrt{1-\frac{9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

iii. Given: For the sphere, radius (r) = 3.5 cm

To Find: Surface area of the sphere.

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 88 \times 0.5 \times 3.5$

$= 154\ cm^2$

$\therefore$ The surface area of sphere is $154\ cm^2$

(ii) Solve any four of the following sub-questions :

i. $m(\text{arc}\, PQR) = 180^\circ$ ... [measure of semicircle]

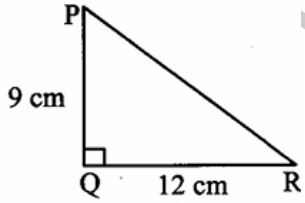
$\therefore m(\text{arc PXR}) = \boxed{180^\circ}$

$\therefore \angle PQR = \frac{1}{2}\, m(\text{arc } \boxed{\text{PXR}}\ )$ ... Inscribed angle theorem

$= \frac{1}{2} \times 180^\circ$

$\therefore \angle PQR = \boxed{90^\circ}$

ii.



Let $\triangle PQR$ be the given right angled triangle.

In $\triangle PQR, \angle Q = 90^\circ$

$\therefore PR^2 = PQ^2 + QR^2$ ... [Pythagoras theorem]

$= 9^2 + 12^2$

$= 81 + 144$

$= 225$

$\therefore PR = \sqrt{225}$ ...[Taking square root of both sides]

= 15 cm

$\therefore$ The length of the hypotenuse of the right angled triangle is 15 cm.

iii. In $\square ABCD$,

$\angle CAD = \angle CBD = \boxed{90}^\circ$ ...[Tangent theorem]

$\therefore \angle ACB + \angle CAD + \angle CBD + \angle ADB = \boxed{360}^o$

$\therefore 52^\circ + 90^\circ + 90^\circ + \angle ADB = 360^\circ$

$\therefore \angle ADB + \boxed{232}^\circ = 360^\circ$

$\angle ADB = 360^\circ - 232^\circ$

$\therefore \angle ADB = \boxed{128}^o$

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iv. $A(x_1, y_1) = A(2, 3)$ and $B(x_2, y_2) = B(4, 7)$

Here, $x_1 = 2, x_2 = 4, y_1 = 3, y_2 = 7$

Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2} = \frac{4}{2} = 2$

$\therefore$ The slope of line AB is 2.

v. Given: Radius (r) = 7cm, Length of arc (l) = 20 cm

To find: Area of the sector

Area of sector $= \frac{l \times r}{2}$

$= \frac{20 \times 7}{2}$

$= 10 \times 7 = 70\ cm^2$

$\therefore$ The area of the sector is $70\ cm^2$.

3. (i) Complete any one activity of the following and rewrite it :

i. In $\triangle PMQ$,

Ray MX is the bisector of $\angle PMQ$

$\therefore \frac{MP}{MQ} = \frac{\boxed{PX}}{\boxed{XQ}}$ ...(i) [Theorem of angle bisector]

Similarly, in $\triangle PMR$, Ray MY is bisector of $\angle PMR$

$\therefore \frac{MP}{MR} = \frac{\boxed{PY}}{\boxed{YR}}$ ...(ii) [Theorem of angle bisector]

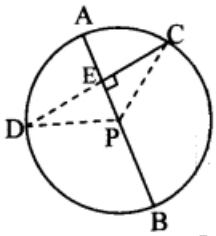
But $\frac{MP}{MQ} = \frac{MP}{MR}$ ...(iii) [As M is the midpoint of QR]

Hence $MQ = MR$

$\therefore \frac{PX}{\boxed{XQ}} = \frac{\boxed{PY}}{YR}$ ...[From (i), (ii) and (iii)]

$\therefore XY \| QR$ ...[Converse of basic proportionality theorem]

ii. Proof:



$CE \perp AB$ ...[Given]

$\therefore DE \perp AB$ ...[C - E - D]

P is the centre of circle.

Then, $PC = PD$ ...(i)[Radii of the circle]

In $\triangle PEC$,

$PC^2 = CE^2 + PE^2 \ldots$[By Pythagoras theorem]

$\therefore \quad \left.\begin{array}{l} \mathrm{CE}^2 = \mathrm{PC}^2 - \mathrm{PE}^2 \\ \text{Similarly }, \mathrm{DE}^2 = \mathrm{PD}^2 - \mathrm{PE}^2 \end{array}\right\}$ ...(ii)

$\therefore CE^2 = DE^2$ ...[From (i) and (ii)]

$\therefore CE = DE$ ...(iii)

By using theorem of intersection of chords,

$AE \times BE = CE \times DE$

$\therefore AE \times BE = CE \times CE$ ...[From (iii)]

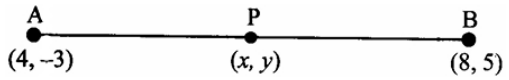
$\therefore AE \times BE = CE^2$

$\therefore CE = \sqrt{AE \times BE}$

$\therefore$ CE is the geometric mean of AE and EB.

(ii) Solve any two of the following sub-questions :

i.



$\therefore$ By section formula,

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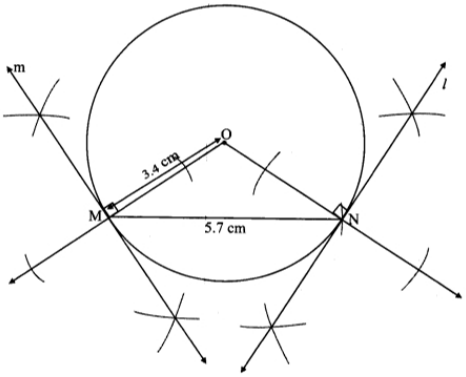
$x = \frac{mx_2+nx_1}{\boxed{m+n}}, y = \frac{\boxed{my_2+ny_1}}{m+n}$

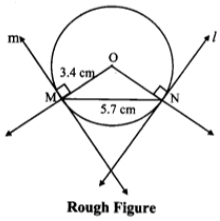
$\therefore x = \frac{3\times8+1\times4}{3+1}, y = \frac{3\times5+1\times(-3)}{3+1}$

$= \frac{\boxed{24}+4}{4} = \frac{\boxed{15}-3}{4}$

$\therefore x = \boxed{7} \therefore y = \boxed{3}$

ii.





Rough Figure

Steps of construction:

i. With centre O, draw a circle of radius 3.4 cm.

ii. Draw chord MN of length 5.7 cm in the circle.

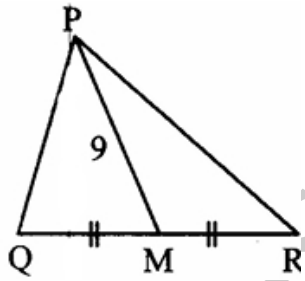
iii. Draw rays OM and ON.

iv. Draw line $l \perp$ ray ON at point N.

v. Draw line $m \perp$ ray OM at point M.

Lines l and m are the required tangents at points M and N to the circle.

iii. In $\triangle PQR$, PM is median,



$PM = 9$ and $PQ^2 + PR^2 = 290$ ...[Given]

$\therefore PQ^2 + PR^2 = 2PM^2 + 2QM^2$ ...[Apollonius theorem]

$290 = 2\left[PM^2 + QM^2\right]$

$\frac{290}{2} = 9^2 + QM^2$

$\therefore 145 = 81 + QM^2$

$\therefore 145 - 81 = QM^2$

$\therefore QM^2 = 64$

$\therefore QM = \sqrt{64} \ldots$ [Taking square root of both sides]

$\therefore QM = 8\ cm$

Now. $QR = 2 \times QM \ldots [M$ is midpoint of $QR]$

$= 2 \times 8$

$\therefore QR = 16\ cm$

4. Solve any two of the following sub-questions :

(i) Let AB be the tower of height 48 m.

Let C and D be the position of two cars at angles of depression 60° and 30° respectively.

AM is the horizontal line and $\angle MAD$ and $\angle MAC$ are angles of depression of measures 30° and 60° respectively.

$\angle MAD$ and $\angle ADB$, $\angle MAC$ and $\angle ACB$ are pairs of alternate angles.

$\therefore m\angle ADB = 30^\circ$ and $m\angle ACB = 60^\circ$

To find: Distance between C and D.

i.e., $l(\text{seg}\, CD)$

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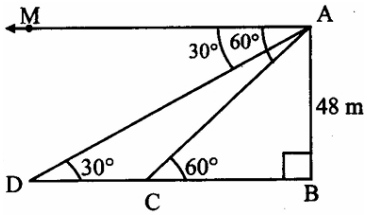


In $\triangle ADB, \angle B = 90^\circ$

$\therefore \tan 30^\circ = \frac{AB}{DB}$

$\therefore \frac{1}{\sqrt{3}} = \frac{48}{DB}$

$\therefore DB = 48\sqrt{3}\ cm$

In $\triangle ACB, \angle B = 90^\circ$

$\therefore \tan 60^\circ = \frac{AB}{CB}$

$\therefore \sqrt{3} = \frac{48}{CB}$

$\therefore CB = \frac{48}{\sqrt{3}}$

$\therefore CB = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ...[Multiply numerator and denominator by $\sqrt{3}$]

$\therefore CB = \frac{48\sqrt{3}}{3}$

$\therefore CB = 16\sqrt{3}\ cm$

$\therefore DC = DB - CB$

$= 48\sqrt{3} - 16\sqrt{3}$

$= 32\sqrt{3}$

$= 32 \times 1.73 \ldots [\sqrt{3} = 1.73]$

$= 55.36\ cm$

$\therefore$ The distance between the two cars is $55.36\ cm$.

(ii) For water:

Radius (r) $= 12\ cm$

Height (h) $= 20\ cm$

Height of the water after dropping the spherical iron ball is

$h_1 = h + 6.75 = 20 + 6.75$

$\therefore h_1 = 26.75\ cm$

Let radius of the spherical iron ball be R cm.

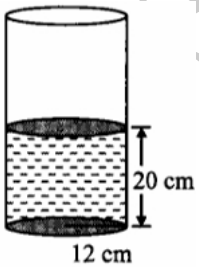
Volume of the water

$= \pi r^2 h$

$= \pi \times (12)^2 \times 20$

$= \pi \times 144 \times 20$

$= 2880\pi\ cm^2$ ...(i)



Volume of the spherical iron ball

$= \frac{4}{3}\pi R^3$ ...(ii)

Note that total volume of the water and the spherical iron ball $= \pi r^2 h_1$

$= \pi(12)^2 \times 26.75$

$= \pi \times 144 \times 26.75$

$= 3852\pi\ cm^2$ ...(iii)

$\therefore$ From (i), (ii) and (iii), we get

$2880\pi + \frac{4}{3}\pi R^3 = 3852\pi$

$\therefore \frac{4}{3}R^3 = 3852 - 2880$ ...[Dividing both sides by π]

$\therefore \frac{4}{3}R^3 = 972$

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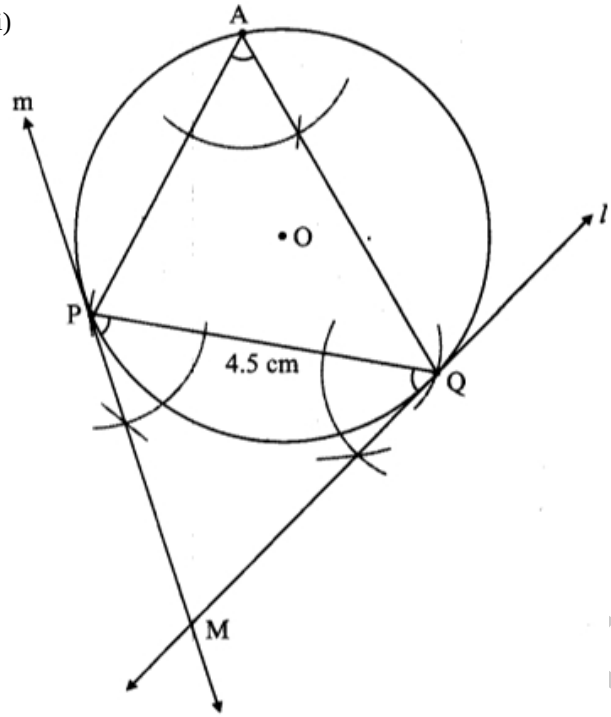
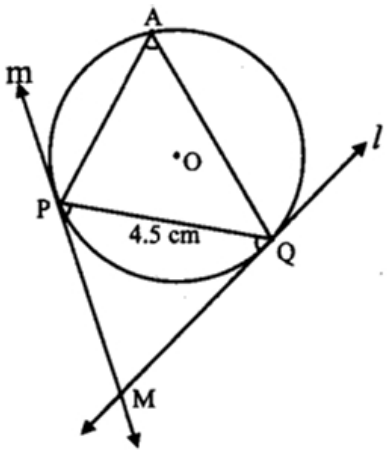
$\therefore R^3 = \frac{972 \times 3}{4} = 729$

$\therefore R = 9 \ldots$ [Taking cube root of both sides]

$\therefore$ Radius of the iron ball is 9 cm.

(iii)





Rough Figure

Steps of construction:

i. Draw a circle of radius 2.7 cm.

ii. Draw a chord PQ of length 4.5 cm.

iii. Take a point A on the major arc, other than P and Q.

iv. Join PA and QA.

v. Using P and Q as vertices and chord PQ as one side, draw $\angle MPQ$ and $\angle MQP$ equal to $\angle PAQ$.

vi. Lines containing the rays PM and QM are the tangents to the circle at P and Q respectively.

5. Solve any one of the following sub-questions :

(i) In $\triangle XDE, PQ \| DE$ ...[Given]

$\therefore \frac{XP}{PD} = \frac{\boxed{XQ}}{QE} \ldots$ (i) [Basic proportionality theorem]

In $\triangle XEF, QR \| EF \ldots$ [Given]

$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ ...(ii) Basic proportionality theorem

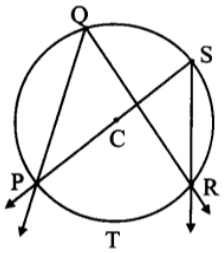
$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ ...[From (i) and (ii)]

$\therefore$ seg $PR \|$ seg DF ... [By converse of basic proportionality theorem]

(ii) Given: C is the centre of circle.

$\angle PQR$ and $\angle PSR$ are inscribed in same arc PTR.

To Prove: $\angle PQR \cong \angle PSR$



Proof:

arc PTR is intercepted by $\angle PQR$.

arc PTR is intercepted by $\angle PSR$.

$\angle PQR = \frac{1}{2}\, m(\text{arc}\, PTR)$, and ...(I) [Inscribed angle theorem]

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$\angle PSR = \frac{1}{2}\ m(\text{arc}\, PTR)$ ...(ii) [Inscribed angle theorem]

$\therefore \angle PQR \cong \angle PSR$ ...[From (i) and (ii)]

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