Solution

GEOMERTY

Class 10 - Mathematics - II

1. (i) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

   i. **(c)** 40 cm

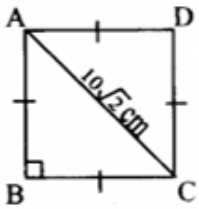
   Explanation:

   In $\triangle ABC$,

   $\angle B = 90^\circ$, and

   $\angle BAC = \angle BCA = 45^\circ$

   $\therefore AB = \frac{1}{\sqrt{2}} AC$ ...[Theorem of $45^\circ - 45^\circ - 90^\circ$ triangle]

   $= \frac{1}{\sqrt{2}} \times 10\sqrt{2}$

   $\therefore AB = 10\ cm$

   $\therefore$ Perimeter of square $= 4(AB) = 4 \times 10 = 40\ cm$

   ii. **(b)** $\sec^2\theta$

   Explanation:

   $\sec^2\theta$

   iii. **(a)** $\frac{1}{\sqrt{3}}$

   Explanation:

   Angle made with the positive direction of X -axis $(\theta) = 30^\circ$

   Slope of line $(m) = \tan\theta$

   $\therefore m = \tan 30^\circ = \frac{1}{\sqrt{3}}$

   iv. **(b)** one

   Explanation:

   one

(ii) Solve the following sub-questions :

i. Let the corresponding sides of similar triangles be s_1 and s_2.

Let A_1 and A_2 be their corresponding areas.

$s_1 : s_2 = 3 : 5$ ...[Given]

$\therefore \frac{s_1}{s_2} = \frac{3}{5}$ ...(i)

$\frac{A_1}{A_2} = \frac{{s_1}^2}{{s_2}^2}$ ...[Theorem of areas of similar triangles]

$= \left(\frac{s_1}{s_2}\right)^2$

$= \left(\frac{3}{5}\right)^2$ ...[From (i)]

$= \frac{9}{25}$

$\therefore$ Ratio of areas of similar triangles $= 9 : 25$

ii. Note that, the distance between the centres of the circles touching externally is equal to the sum of their radii.

$\therefore$ Required distance $= 5 + 3 = 8\ cm$

iii. In $\triangle PSR, \angle S = 90^\circ, \angle P = 30^\circ$ . . . [Given]

$\therefore \angle R = 60^\circ$ . . . [Remaining angle of a triangle]

$\therefore \triangle PSR$ is a $30^\circ - 60^\circ - 90^\circ$ triangle.

$RS = \frac{1}{2} RP$ . . .[Side opposite to 30°]

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$\therefore 6 = \frac{1}{2}RP$

$\therefore RP = 6 \times 2$

$\therefore RP = 12$ units

iv. Here, $x_1 = 2, x_2 = 4, y_1 = 3, y_2 = 7$

Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2} = \frac{4}{2} = 2$

2. (i) Complete any two activities and rewrite it :

i. In $\square ABCD$,

$\angle CAD = \angle CBD = \boxed{90}^\circ$ ...[Tangent theorem]

$\therefore \angle ACB + \angle CAD + \angle CBD + \angle ADB = \boxed{360}^o$

$\therefore 52^\circ + 90^\circ + 90^\circ + \angle ADB = 360^\circ$

$\therefore \angle ADB + \boxed{232}^\circ = 360^\circ$

$\angle ADB = 360^\circ - 232^\circ$

$\therefore \angle ADB = \boxed{128}^o$

ii. In $\triangle ABC, \angle ABC = 90^\circ, \angle C = \theta^\circ$.

$AB^2 + BC^2 = \boxed{AC^2}$ ...[Pythagoras theorem]

Divide both sides by AC^2

$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$

$\therefore \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$

But $\frac{AB}{AC} = \boxed{\sin\theta}$ and $\frac{BC}{AC} = \boxed{\cos\theta}$

$\therefore \sin^2\theta + \cos^2\theta = \boxed{1}$

iii. Area of square $= \boxed{(\text{side})^2}$ ...[Formula]

$= (7)^2$

$= 49\ cm^2$

Area of sector (D-AXC)

$= \boxed{\frac{\theta}{360} \times \pi r^2}$ ...[Formula]

$= \frac{\boxed{90}}{360} \times \frac{22}{7} \times \boxed{7^2}$

$= 38.5\ cm^2$

(ii) Solve any four of the following sub-questions :

i. Chords AC and DE intersect internally at point B.

$\therefore \angle ABE = \frac{1}{2}[\ m(\text{arc}\,AE) + m(\text{arc}\,DC)]$

$\therefore 108^\circ = \frac{1}{2}[95^\circ + m(\text{arc}\,DC)]$

$\therefore 108^\circ \times 2 = 95^\circ + m(\text{arc}\,DC)$

$\therefore 95^\circ + m(\text{arc}\,DC) = 216^\circ$

$\therefore m(\text{arcDC}) = 216^\circ - 95^\circ$

$\therefore m(\text{arc}\,DC) = 121^\circ$

ii. In $\triangle ABC, \angle ABC = 90^\circ$

$AD = 4, DC = 9, \text{seg}\,BD \perp \text{seg}\,AC$ ...[Given]

$\therefore BD^2 = AD \times DC$ ...[Theorem of geometric mean]

$\therefore BD^2 = 4 \times 9$

$\therefore BD = \sqrt{36}$

$\therefore BD = 6$ units

iii. i. $m(\text{arc}\,EF) = m\angle ECF$ ...[Definition of measure of minor arc]

$\therefore m(\text{arc}\,EF) = 70^\circ$

$m(\text{arc}\,DE) + m(\text{arc}\,DGF) + m(\text{arc}\,EF) = 360^\circ$ ...[Measure of a circle is 360°]

$\therefore m(\text{arc}\,DE) = 360^\circ - m(\text{arc}\,DGF) - m(\text{arc}\,EF)$

$= 360^\circ - 200^\circ - 70^\circ$

$\therefore m(\text{arc}\,DE) = 90^\circ$

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ii. $m(\text{arc } DEF) = m(\text{arc } DE) + m(\text{arc } EF)$ ...[Arc addition property]

$= 90^\circ + 70^\circ$

$\therefore m(\text{arc DEF}) = 160^\circ$

iv. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the given points.

$\therefore x_1 = -1, y_1 = 1, x_2 = 5, y_2 = -7$

By distance formula,

$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5+1)^2 + (-7-1)^2}$

$= \sqrt{6^2 + (-8)^2}$

$= \sqrt{36 + 64} = \sqrt{100}$

$\therefore d(P,Q) = 10$ units

$\therefore$ The distance between the points P and Q is 10 units.

v. Area of square $ABCD$ = $\boxed{(side)^2}$

$= 14^2 = 196\ cm^2$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 7^2$

$= \boxed{154} cm^2$

Area of shaded portion

= Area of square ABCD - Area of circle

$= 196 - \boxed{154}$

$= \boxed{42}\ cm^2$

3. (i) Complete any one activity of the following and rewrite it :

i. In $\triangle XDE, PQ \| DE$ ...[Given]

$\therefore \frac{XP}{\boxed{PD}} = \frac{\boxed{XQ}}{QE}$ ...(i) [Basic proportionality theorem]

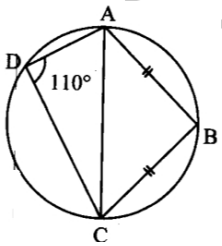
In $\triangle XEF, QR \| EF$ ...[Given]

$\therefore \frac{XQ}{QE} = \frac{\boxed{XR}}{RF}$ ... (ii) Basic proportionality theorem

$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ From (i) and (ii)]

$\therefore$ seg PR $\|$ seg DF ...[Converse of basic proportionality theorem]

ii. i.



ii. $\square ABCD$ is a cyclic quadrilateral. ...[Given]

$\therefore \angle ADC + \angle ABC = 180^\circ$ ...[Opposite angles of a cyclic quadrilateral are supplementary]

$\therefore 110^\circ + \angle ABC = 180^\circ$

$\therefore \angle ABC = 180^\circ - 110^\circ$

$\therefore m\angle ABC = 70^\circ$

iii. In $\triangle ABC$,

side $AB \cong$ side BC ...[Given]

$\therefore \angle ACB \cong \angle BAC \ldots$ [Isosceles triangle theorem]

Let $\angle ACB = \angle BAC = x$

Now, $\angle ABC + \angle BAC + \angle ACB = 180^\circ \cdots$ [Sum of the measures of angles of a triangle is 180°]

$\therefore \angle ABC + x + x = 180^\circ$

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$\therefore 70^\circ + 2x = 180^\circ$

$\therefore 2x = 180^\circ - 70^\circ$

$\therefore 2x = 110^\circ$

$\therefore x = \frac{110^\circ}{2} = 55^\circ$

$\therefore m\angle BAC = 55^\circ$ ...(I)

iv. $\angle ADC = \frac{1}{2}\, m(\text{arc } ABC)$ ...[Inscribed angle theorem]

$\therefore 110^\circ = \frac{1}{2}\, m(\text{arc ABC})$

$\therefore m(\text{arc } ABC) = 220^\circ$

(ii) Solve any two of the following sub-questions :

i. Let $A(x_1, y_1) = A(4, 7)$,

$B(x_2, y_2) = B(8, 4)$

$C(x_3, y_3) = C(7, 11)$

$\therefore$ By centroid formula,

$x = \frac{x_1+x_2+x_3}{3} = \frac{4+8+7}{3} = \frac{19}{3}$

$y = \frac{y_1+y_2+y_3}{3} = \frac{7+4+11}{3} = \frac{22}{3}$

$\therefore$ The co-ordinates of the centroid are $\left(\frac{19}{3}, \frac{22}{3}\right)$.

ii. In $\triangle OAP$,

seg $OA \perp$ tangent AP ... [Tangent is perpendicular to radius]

$\therefore \angle OAP = 90^\circ$

$\therefore OP^2 = OA^2 + AP^2$ ... [Pythagoras theorem]

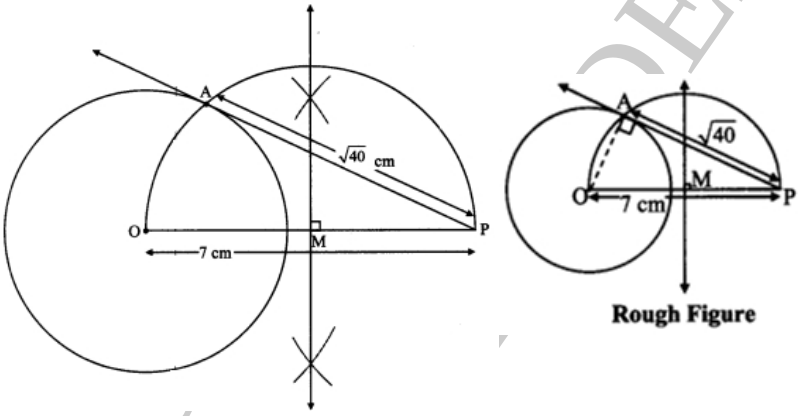
$\therefore OP^2 = (3)^2 + (\sqrt{40})^2$

$\therefore OP^2 = 9 + 40$

$\therefore OP^2 = 49$

$\therefore OP = 7\ cm$

$\therefore$ Point P is at a distance 7 cm from the centre.



Steps of construction:

i. Draw a circle with centre O and radius 3 cm.

ii. Take a point P such that $OP = 7\ cm$.

iii. Obtain the midpoint M of segment OP by drawing the perpendicular bisector.

iv. Draw a semicircle with radius OM and centre M.

v. Let A be the point of intersection of the semicircle and the circle with centre O.

vi. Draw a line joining points A and P.

Line AP is the required tangent.

iii. According to the given conditions, in $\triangle ABC$, altitude $AM = 2\sqrt{3}$ units and in $\triangle PQR$, altitude $PN = 4\sqrt{3}$ units. Also, $\triangle ABC$ and are equilateral triangles.

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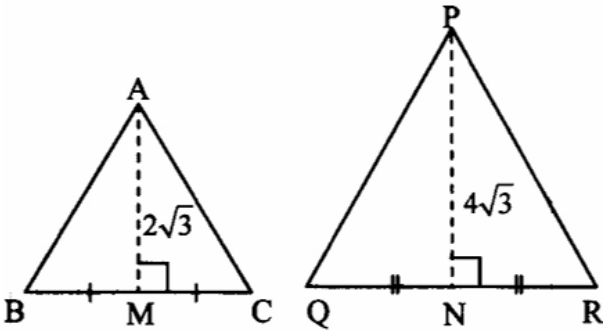


i. In $\triangle ABM, \angle ABM = 60^\circ, \angle AMB = 90^\circ$

$\therefore \angle MAB = 30^\circ$ ...[Remaining angle of a triangle]

$\therefore \triangle ABM$ is a $30^\circ - 60^\circ - 90^\circ$ triangle.

$AM = \frac{\sqrt{3}}{2} AB$ ...[Side opposite to 60°]

$\therefore 2\sqrt{3} = \frac{\sqrt{3}}{2} AB$

$\therefore AB = 4$ units

Similarly, we get

$PN = \frac{\sqrt{3}}{2} PQ$ ...[Side opposite to 60°]

$\therefore 4\sqrt{3} = \frac{\sqrt{3}}{2} PQ$

$\therefore PQ = 8$ units

ii. $\triangle ABC \sim \triangle PQR \ldots$ [Equilateral triangles]

$\therefore \frac{A(\triangle ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...[Theorem of areas of similar triangles]

$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{4^2}{8^2} = \frac{16}{64} = \frac{1}{4}$

iii. $\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{3 \times AB}{3 \times PQ} = \frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2}$

$\therefore$ Required ratio is $1 : 2$.

4. Solve any two of the following sub-questions :

(i) $\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\tan^2\theta} - \frac{1}{\cot^2\theta} - \frac{1}{\sec^2\theta} - \frac{1}{\text{cosec}^2\theta} = -3$

$\therefore \frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)} - \frac{1}{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)} - \cos^2\theta - \sin^2\theta = -3$

$\therefore \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} - (\cos^2\theta + \sin^2\theta) = -3$

$\therefore \frac{1-\cos^2\theta}{\sin^2\theta} - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3 \ \ldots \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\therefore \frac{\sin^2\theta}{\sin^2\theta} - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3 \ \ldots \left[\because 1 - \cos^2\theta = \sin^2\theta\right]$

$\therefore 1 - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3$

$\therefore -\frac{1+\sin^2\theta}{1-\sin^2\theta} = -3$

$\therefore \frac{1+\sin^2\theta}{1-\sin^2\theta} = 3$ ...[Multiplying both sides by -1]

$\therefore 1 + \sin^2\theta = 3 - 3\sin^2\theta$

$\therefore \sin^2\theta + 3\sin^2\theta = 3 - 1$

$\therefore 4\sin^2\theta = 2$

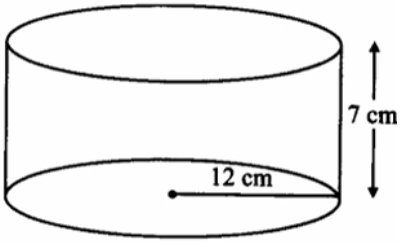
$\therefore \sin^2\theta = \frac{2}{4}$ ...[Dividing both sides by 4]

$\therefore \sin^2\theta = \frac{1}{2}$

$\therefore \sin\theta = \frac{1}{\sqrt{2}}$ ...[Taking square root of both sides]

$\therefore \theta = 45^\circ$

(ii)



For cylindrical shaped ice-cream pot,

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Height $(H) = 7\ cm$ and Radius $(R) = 12\ cm$.

Volume of cylindrical shaped ice-cream pot

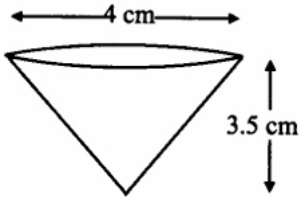
$= \pi R^2 H$

$= \frac{22}{7} \times (12)^2 \times 7$

$= 22 \times 144$

$= 3168\ cm^3$

$\therefore$ The quantity of the ice-cream filled in the pot is $3168\ cm^3$.



For right circular ice-cream cones,

eight $(h) = 3.5\ cm$ and Radius $(r) = \frac{4}{2} = 2\ cm$.

Volume of right circular ice-cream cone

$= \frac{1}{3}\pi r^2\ h$

$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 3.5$

$= \frac{1}{3} \times 22 \times 4 \times 0.5$

$= \frac{44}{3}\ cm^3$

$\therefore$The quantity of the ice-cream filled in one ice-cream cone is $\frac{44}{3}\ cm^3$.

Number of students served

$=$ (Quantity of ice-cream in the pot) $\div$ (Quantity of ice-cream in one cone)

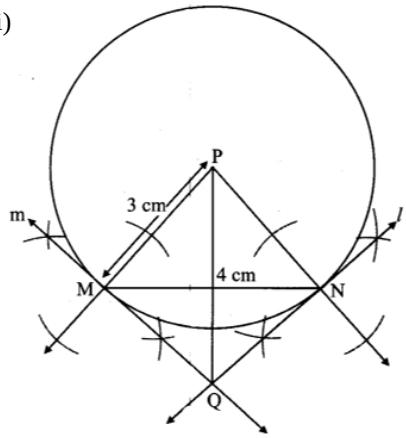
$= 3168 \div \frac{44}{3}$

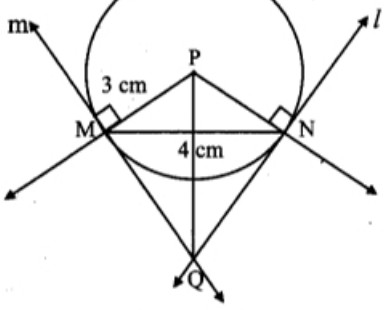
$= 3168 \times \frac{3}{44}$

$= 216$

$\therefore$ 216 students can be served.

(iii)





Rough Figure

$\therefore l(PQ) = 4\ cm$

Steps of construction:

i. With centre P, draw a circle of radius 3 cm.

ii. Draw chord MN of length 4 cm in the circle.

iii. Draw rays PM and PN.

iv. Draw line $l \perp$ ray PN at point N.

v. Draw line $m \perp$ ray PM at point M.

vi. Lines l and m intersect at point Q.

vii. Lines l and m are the required tangents at points M and N to the circle.

viii. Measure the length of PQ.

5. Solve any one of the following sub-questions :

(i) Given: line $l \parallel$ line $m \parallel$ line n

Transversals t_1 and t_2 intersect the parallel lines at points A, B, C and P, Q, R respectively.

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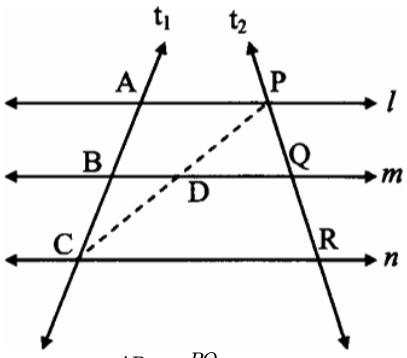

To prove: $\frac{AB}{BC} = \frac{PQ}{QR}$

Construction: Draw seg PC which intersects line m at point D.

Proof: In $\triangle ACP$, seg $BD \| $ seg AP ...[Given]

$\therefore \frac{AB}{BC} = \frac{PD}{DC}$ ... (i)[Basic proportionality theorem]

In $\triangle RPC$, seg $DQ \| $ seg CR ...[Given]

$\therefore \frac{PQ}{QR} = \frac{PD}{DC}$ ... (ii)[Basic proportionality theorem]

$\therefore \frac{AB}{BC} = \frac{PQ}{QR}$ ... [From (i) and (ii)]

(ii) $m(\text{arc } ABC) = m(\text{arc } AB) + \boxed{m(\text{arc } BC)}$

$= \boxed{125}^\circ + 110^\circ = 235^\circ$

$m(\text{arc } AC) = 360^\circ - m(\text{arc}\boxed{ABC})$

$= 360^\circ - \boxed{235}^\circ = 125^\circ$

Similarly,

$m(\text{arc } ACB) = 360^\circ - \boxed{125}^\circ$

$= 235^\circ$

and $m(\text{arc BAC}) = 360^\circ - \boxed{110}^\circ = 250^\circ$

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