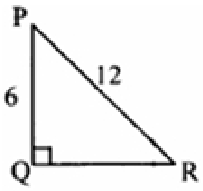
Solution

GEOMERTY

Class 10 - Mathematics - II

1. (i) Four alternative answers for each of the following sub-questions are given. Choose the correct alternative and write its alphabet:

i. **(a)** 60°
Explanation:
$PR = 12$ and $PQ = 6$ ...[Given]
PR is the hypotenuse,
$\therefore \angle Q = 90^\circ$
Now, $\cos P = \frac{PQ}{PR}$
$\therefore \cos P = \frac{6}{12}$
$\cos P = \frac{1}{2}$ ...(i)
we know, $\cos 60^\circ = \frac{1}{2}$ ...(ii)

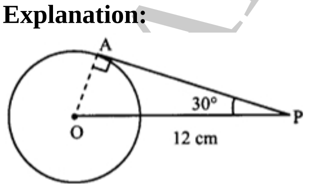


From equation (i) and (ii)
$\cos P = \cos 60^\circ$
$\therefore \angle P = 60^\circ$

ii. **(b)** $\sec^2 \theta$
Explanation:
$\sec^2 \theta$

iii. **(d)** $(-5, 3)$
Explanation:
According to the given conditions,
Y-coordinate of the point B must be 3.

iv. **(a)** 6 cm
Explanation:



In $\triangle AOP$,
$\sin 30^\circ = \frac{AO}{OP}$
$\therefore \frac{1}{2} = \frac{AO}{12}$
$\therefore AO = 6\ cm$

(ii) Solve the following sub-questions :

i. $\triangle ABC$ and $\triangle ADB$ have same base AB.
$\therefore \frac{A(\triangle ABC)}{A(\triangle ADB)} = \frac{BC}{AD}$ ...[Triangles having equal base]
$= \frac{4}{8}$
$\therefore \frac{A(\Delta ABC)}{A(\triangle ADB)} = \frac{1}{2}$

ii. As opposite angles of a cyclic quadrilateral are supplementary,
$\angle B + \angle D = 180^\circ$
$\therefore 75^\circ + \angle D = 180^\circ$
$\therefore \angle D = 180^\circ - 75^\circ$
$\therefore \angle D = 105^\circ$

iii. In $\triangle ABC, \angle ABC = 90^\circ, \angle BAC = \angle BCA = 45^\circ$ ...[Given]

$\therefore AB = \frac{1}{\sqrt{2}} AC \ldots [By 45^\circ - 45^\circ - 90^\circ$ Theorem]

$\therefore AB = \frac{1}{\sqrt{2}} \times 9\sqrt{2}$

$\therefore AB = 9$ units

iv. Angle made with the positive direction of $X - \text{axis}(\theta) = 45^\circ$

Slope of the line $(m) = \tan\theta$

$\therefore m = \tan 45^\circ = 1$

$\therefore$ The slope of the line is 1.

2. (i) Complete any two activities and rewrite it :

i. Ray PQ is a tangent to the circle at point Q and seg PS is the secant. ...[Given]

$\therefore PR \times PS = PQ^2$ ...[Tangent secant segments theorem]

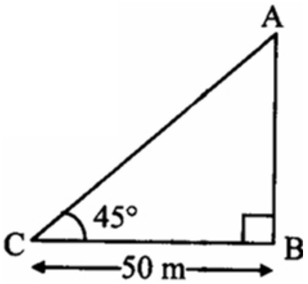
$\therefore 8 \times PS = 12^2$

$\therefore 8 \times PS = 144$

$\therefore PS = \frac{144}{8}$

$\therefore$ PS = 18 units

ii. Let AB represent the height of the temple and point C represent the position of the person.



$BC = 50\ m$

Angle of elevation $= \angle ACB = 45^\circ$

In right angled $\triangle ABC$,

$\tan 45^\circ = \frac{AB}{BC}$ ...[By definition]

$\therefore 1 = \frac{AB}{50}$

$\therefore AB = 50\ m$

$\therefore$ The height of the temple is $50\ m$.

iii. Number of cylinders can be made

$= \frac{\text{Volume of the sphere}}{\boxed{\text{Volume of a cylinder}}}$

$= \frac{\frac{4}{3}\pi r^3}{\boxed{\pi R^2 H}}$

$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{\boxed{6 \times 6 \times 12}}$

$= \boxed{18}$

(ii) Solve any four of the following sub-questions :

i. Line KL is the tangent to the circle at point L and seg ML is the radius.

... [Given]

$\therefore \angle MLK = 90^\circ$ (i) ... [Tangent theorem]

In $\triangle MLK, \angle MLK = 90^\circ$

$\therefore MK^2 = ML^2 + KL^2 \ldots$ [Pythagoras theorem]

$\therefore 12^2 = ML^2 + (6\sqrt{3})^2$

$\therefore 144 = ML^2 + 108$

$\therefore ML^2 = 144 - 108$

$\therefore ML^2 = 36$

$\therefore ML = \sqrt{36} = 6$ units ... [Taking square root of both sides]

$\therefore$ Radius of the circle is 6 units.

ii. Here , $AB = 9\ cm, BC = 40\ cm, AC = 41\ cm$

$\therefore 41^2 = 1681$

$9^2 + 40^2 = 1681$

i.e., $AB^2 + BC^2 = AC^2$

$\therefore (9, 40, 41)$ is a Pythagorean triplet.

$\therefore \triangle ABC$ is a right angled triangle.

iii. $\angle ABC = \frac{1}{2}\ m \boxed{(arc AXC)}$ ... [Inscribed angle theorem]

$65^\circ \times 2 = m(\text{arc AXC})$

$m(\text{arc}\ AXC) = \boxed{130^\circ}$

$\angle AOC = m(\text{ arc } AXC)$ ... [Definition of measure of an arc]

$\angle AOC = \boxed{130^\circ}$

iv. Let $P(x_1, y_1) = P(3, -5)$

$Q(x_2, y_2) = Q(4, 3)$

$R(x_3, y_3) = R(11, -4)$

Let $G(x, y)$ be the centroid.

$\therefore$ By centroid formula,

$x = \frac{x_1+x_2+x_3}{3} = \frac{3+4+11}{3} = \frac{18}{3}$

$\therefore x = 6$

$y = \frac{y_1+y_2+y_3}{3} = \frac{-5+3-4}{3} = \frac{-6}{3}$

$\therefore y = -2$

$\therefore$ The coordinates of the centroid are $(6, -2)$.

v. Given: For the sphere, radius (r) = 7 cm

To find: Surface area of the sphere.

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (7)^2$

$= 88 \times 7$

$= 616\ cm^2$

$\therefore$ The surface area of the sphere is $616 cm^2$.

3. (i) Complete any one activity of the following and rewrite it :

i. In $\triangle XDE, PQ \| DE$ ...[Given]

$\therefore \frac{XP}{PD} = \frac{\boxed{XQ}}{QE}$ ... (i) [Basic proportionality theorem]

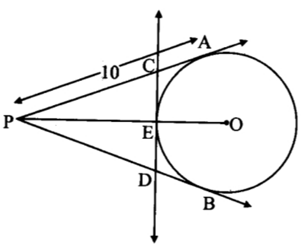
In $\triangle XEF, QR \| EF$ ... [Given]

$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ ...(ii) Basic proportionality theorem

$\therefore \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ ...[From (i) and (ii)]

$\therefore$ seg $PR \|$ seg DF ... [By converse of basic proportionality theorem]

ii. i.



ii. Given that,

seg PA and seg PB are tangents to the circle from point P.

$\therefore$ seg $PA \cong$ seg PB ...[Tangent segment theorem]

iii. Note that $PA = PB = 10$ units

Also, seg CE and seg CA are the tangents to the circle from point C .

$\therefore$ seg $CE \cong$ sec CA ...(i)[Tangent segment theorem]

Here, $PA = PC + CA$

$\therefore PA = PC + CE$ ...(ii)[From (i)]
Similarly, we get
$PB = PD + DE$ ...(iii)
Perimeter of $\triangle PCD$
$= (PC + CE) + (DE + PD)$
$= PA + PB$ ...[From (ii) and (iii)]
$= 10 + 10$
$= 20$ units

(ii) Solve any two of the following sub-questions :

i. Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of line $AB = \frac{2-1}{8-6} = \boxed{\frac{1}{2}}$ ...(i)

$\therefore$ Slope of line $BC = \frac{4-2}{9-8} = \boxed{2}$ ...(ii)

$\therefore$ Slope of line $CD = \frac{3-4}{7-9} = \boxed{\frac{1}{2}}$ ...(iii)

$\therefore$ Slope of line $DA = \frac{3-1}{7-6} = \boxed{2}$ ...(iv)

$\therefore$ Slope of line $AB = \boxed{Slope\ of\ line\ CD}$ ...[From (i) and (iii)]

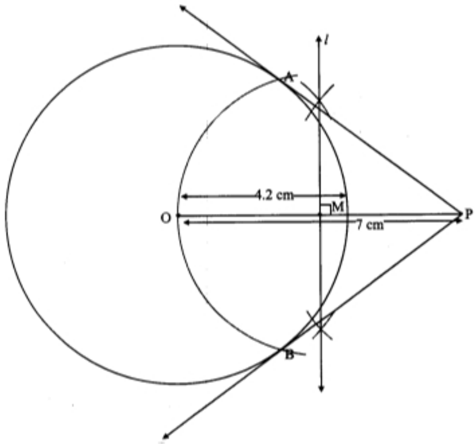
$\therefore$ line $AB||$ line CD

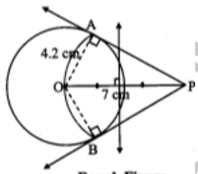
$\therefore$ Slope of line BC $= \boxed{Slope\ of\ line\ DA}$ ...[From (ii) and (iv)]

$\therefore$ line $BC \|$ line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore \square ABCD$ is a parallelogram.

ii. 

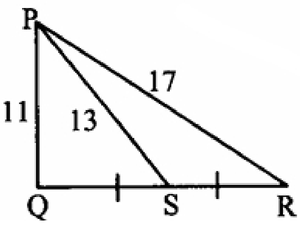


Rough Figure

Steps of construction:

i. With centre O, draw a circle of radius 4.2 cm.
ii. Take point P such that $OP = 7\ cm$.
iii. Draw the perpendicular bisector of seg OP. It intersects OP in point M.
iv. With M as centre and radius equal to OM, draw an arc intersecting the circle in points A and B.
v. Draw rays PA and PB.

Rays PB and PB are the required tangents to the circle.

iii. 

In $\triangle PQR$, point S is the midpoint of side QR.

$\therefore$ seg PS is the median. ...[Given]

$\therefore PQ^2 + PR^2 = 2PS^2 + 2SR^2$ ...[Apollonius theorem]

$\therefore 11^2 + 17^2 = 2(13)^2 + 2$ SR ^{2}

$\therefore 121 + 289 = 2(169) + 2SR^2$

$\therefore 410 = 338 + 2SR^2$
$\therefore 2SR^2 = 410 - 338$
$\therefore 2SR^2 = 72$
$\therefore SR^2 = \frac{72}{2} = 36$
$\therefore SR = \sqrt{36} \ldots$ [Taking square root of both sides]
$= 6$ units
Now, $QR = 2SR \ldots [S$ is the midpoint of QR$]$
$= 2 \times 6$
$\therefore QR = 12$ units

4. Solve any two of the following sub-questions :

(i) $\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\tan^2\theta} - \frac{1}{\cot^2\theta} - \frac{1}{\sec^2\theta} - \frac{1}{\operatorname{cosec}^2\theta} = -3$

$\therefore \frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)} - \frac{1}{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)} - \cos^2\theta - \sin^2\theta = -3$

$\therefore \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} - \left(\cos^2\theta + \sin^2\theta\right) = -3$

$\therefore \frac{1-\cos^2\theta}{\sin^2\theta} - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3 \ \ldots \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\therefore \frac{\sin^2\theta}{\sin^2\theta} - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3 \ \ldots \left[\because 1 - \cos^2\theta = \sin^2\theta\right]$

$\therefore 1 - \frac{1+\sin^2\theta}{\cos^2\theta} - 1 = -3$

$\therefore -\frac{1+\sin^2\theta}{1-\sin^2\theta} = -3$

$\therefore \frac{1+\sin^2\theta}{1-\sin^2\theta} = 3$ ...[Multiplying both sides by -1]

$\therefore 1 + \sin^2\theta = 3 - 3\sin^2\theta$

$\therefore \sin^2\theta + 3\sin^2\theta = 3 - 1$

$\therefore 4\sin^2\theta = 2$

$\therefore \sin^2\theta = \frac{2}{4}$ ...[Dividing both sides by 4]

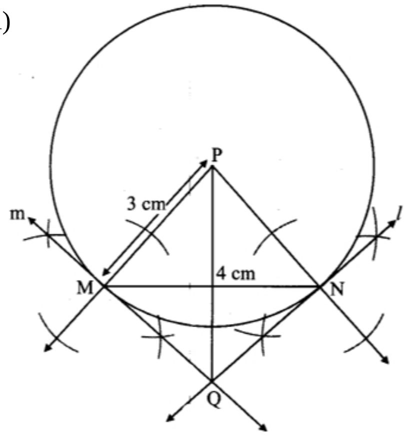
$\therefore \sin^2\theta = \frac{1}{2}$

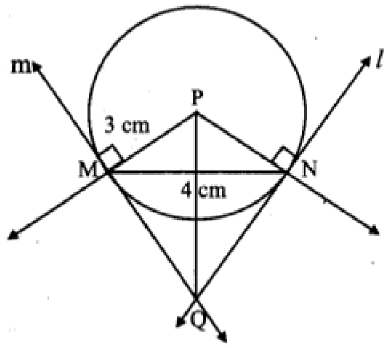
$\therefore \sin\theta = \frac{1}{\sqrt{2}}$ ...[Taking square root of both sides]

$\therefore \theta = 45^\circ$

(ii) Area of square $ABCD = (\text{side})^2$
$= (50)^2$
$= 2500\ m^2$
Radius of sector $A - SP = \frac{1}{2} \times 50 = 25\ m$
$\theta = 90^\circ$ ...[Angle of a square]
Area of sector $A - SP = \frac{\theta}{360} \times \pi r^2$
$= \frac{90}{360} \times \frac{22}{7} \times (25)^2$
$= \left(\frac{1}{4} \times \frac{13750}{7}\right) m^2$
A(shaded region)
$=$ Area of square $ABCD\ -4($ Area of sector $A - SP)$
$= 2500 - 4\left(\frac{1}{4} \times \frac{13750}{7}\right)$
$= 2500 - \frac{13750}{7}$
$= \frac{17500-13750}{7}$
$= \frac{3750}{7}$
$\approx 535.71\ m^2$
Area of the shaded region is $535.71\ m^2$.

(iii)





Rough Figure

$\therefore l(PQ) = 4\ cm$

Steps of construction:

i. With centre P, draw a circle of radius 3 cm.
ii. Draw chord MN of length 4 cm in the circle.
iii. Draw rays PM and PN.
iv. Draw line $l \perp$ ray PN at point N.
v. Draw line $m \perp$ ray PM at point M.
vi. Lines l and m intersect at point Q.
vii. Lines l and m are the required tangents at points M and N to the circle.
viii. Measure the length of PQ.

5. Solve any one of the following sub-questions :

(i) In $\triangle PMQ$,

Ray MX is the bisector of $\angle PMQ$

$\therefore \frac{MP}{MQ} = \frac{\boxed{PX}}{\boxed{XQ}}$ ...(i) [Theorem of angle bisector]

Similarly, in $\triangle PMR$, Ray MY is bisector of $\angle PMR$

$\therefore \frac{MP}{MR} = \frac{\boxed{PY}}{\boxed{YR}}$ ...(ii) [Theorem of angle bisector]

But $\frac{MP}{MQ} = \frac{MP}{MR}$ ...(iii) [As M is the midpoint of QR]

Hence $MQ = MR$

$\therefore \frac{PX}{\boxed{XQ}} = \frac{\boxed{PY}}{YR}$ ...[From (i), (ii) and (iii)]

$\therefore XY \| QR$ ...[Converse of basic proportionality theorem]

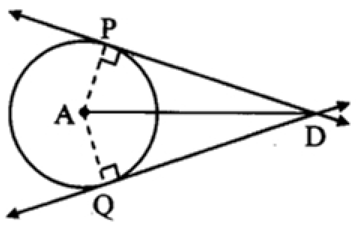
(ii) Given: A is the centre of the circle.

Tangents through external point D touch the circle at the points P and Q.

To prove: seg DP $\cong$ seg DQ.

Construction: Draw seg AP and seg AQ.

Proof:



In $\triangle PAD$ and $\triangle QAD$,

seg $PA \cong$ seg $QA \ldots$ [Radii of the same circle]

seg $AD \cong$ seg $AD \ldots$[Common side]

$\angle APD = \angle AQD = 90^\circ \ldots$ [Tangent theorem]

$\therefore \triangle PAD \cong \triangle QAD \ldots$ [By Hypotenuse side test]

$\therefore$ seg $DP \cong$ seg DQ ...[Corresponding sides of congruent triangles]