Solution

ALGEBRA

Class 10 - Mathematics - I

1. (i) Choose the correct alternative from given :

   i. **(d)** $x(x+5)=2$
   Explanation:
   $x(x+5)=2$

   ii. **(a)** 7
   Explanation:
   $x=\frac{D_x}{D}=\frac{49}{7}=7$

   iii. **(a)** $-2,-4,-6,-8$
   Explanation:
   $a=t_1=-2$
   $d=-2$
   $t_2=t_1+d=-2-2=-4$
   $t_3=t_2+d=-4-2=-6$
   $t_4=t_3+d=-6-2=-8$

   iv. **(d)** 15
   Explanation:
   15

(ii) i.
$$\begin{array}{r} 15x+17y=21 \\ +\ 17x+15y=11 \\ \hline 32x+32y=32 \end{array}$$
$\therefore x+y=1$ ...[Dividing both sides by 32]

ii. The given A.P. is,
$5,12,19,26,\ldots$
Here, first term is 5 and common difference is 7.
$\therefore$ Next two terms in this A.P. are $26+7=33$ and $33+7=40$.
$\therefore$ Next two terms are 33 and 40.

iii. Rate of GST $=12\%$
$\therefore$ Rate of CGST $=$ Rate of SGST
$=\frac{\text{Rate of GST}}{2}=\frac{12}{2}=6\%$
$\therefore$ Rate of CGST $=$ Rate of SGST $=6\%$

iv. **(a)** 1.5
Explanation:
The probability of any event is from 0 to 1 or 0% to 100%.

2. (i) Complete the following activities and rewrite it (any two) :

i. The given A.P. is $9,4,-1,-6,-11,\ldots$
Here, $a=9, d=4-9=-5$
Since $t_n=a+(n-1)d$
$\therefore t_{23}=9+(23-1)(-5)$
$=9+22(-5)$
$=9-110$
$\therefore t_{23}=-101$
$\therefore$ 23rd term of the given A.p. is - 101.

ii. $2y^2-y+2=0$
Comparing the above equation with
$ay^2+by+c=0$, we get
$a=2,\ b=-1, c=2$

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$\therefore b^2 - 4ac = (-1)^2 - 4 \times 2 \times 2$

$= 1 - 16$

$\therefore b^2 - 4ac = -15$

iii. If two coins are tossed simultaneously,

$S = \{HH, HT, TH, TT\}$

i. Event A : at least getting one head.

$\therefore A = \{HH, HT, TH\}$

ii. Event B : to get no head.

$B = \{TT\}$

(ii) Solve the following subquestions (any four) :

i. $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{5} - \boxed{2} \times 4 = \boxed{15} - 8 = \boxed{7}$

ii. Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$

$a = 1,\ b = 10, c = \boxed{-7}$

$\therefore b^2 - 4ac = \boxed{10^2} - 4 \times 1 \times (-7)$

$= 100 + \boxed{28}$

$= \boxed{128}$

iii. First term $= a = 6$, common difference $= d = 3$, $S_{27} = ?$

$S_n = \frac{n}{2}[\boxed{2a} + (n-1)d]$

$\therefore S_{27} = \frac{27}{2}[12 + (27-1)\boxed{3}]$

$= \frac{27}{2} \times \boxed{90} = 27 \times 45$

$\therefore S_{27} = \boxed{1215}$

iv. Total number of pens in the box

$= 5 + 8 + 3 = 16$

$\therefore n(S) = 16$

Let A be the event that Rutuja picks a blue pen.

Total number of blue pens = 8

$\therefore n(A) = 8$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{8}{16}$

$\therefore P(A) = \frac{1}{2}$

$\therefore$ The probability that Rutuja picks a blue pen is $\frac{1}{2}$.

v. $\Sigma f d_i = 320, \Sigma f_i = 50$ ...[Given]

$\bar{d} = \frac{\sum f_d d_i}{\sum f_i} = \frac{320}{50} = 6.40$

Mean $= A + \bar{d}$

$= 300 + 6.40$

$\therefore$ Mean $= 306.40$

3. (i) Complete the following activity and rewrite it (any one) :

i. i. Let the total amount spent on sports be ₹ x.

$\therefore$ The total amount ₹x corresponds to central angle $= 360^\circ$

Central angle for expenditure on football $= 45^\circ$

Central angle for football

$= \frac{\text{Expenditure on football}}{\text{Total expenditure}} \times 360^\circ$

$\therefore 45 = \frac{9000}{x} \times 360$

$\therefore x = \frac{9000 \times 360}{45} = 72000$

$\therefore$ The total amount spent on sports is ₹ $72,000$

ii. Central angle for expenditure on cricket $= 160^\circ$ Central angle for cricket

$= \frac{\text{Expenditure on cricket}}{\text{Total expenditure}} \times 360^\circ$

$\therefore 160 = \frac{\text{Expenditure on cricket}}{72000} \times 360$

$\therefore$ Expenditure on cricket $= \frac{160}{360} \times 72000 = 32000$

$\therefore$ The amount spent on cricket is ₹ 32,000.

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ii. Total value for toy car with GST = ₹ 2360

Rate of GST $= 18\%$

Let taxable value for toy car be ₹x

$\therefore$ GST $= \frac{18}{100} \times x$

$\therefore$ Total value for toy car = (taxable value for toy car) + $\boxed{GST}$

$\therefore 2360 = \boxed{x} + \frac{\boxed{18}}{100} \times x$

$\therefore 2360 = \frac{\boxed{118}}{100} \times x$

$\therefore 2360 \times 100 = 118x$

$\therefore x = \frac{2360 \times 100}{\boxed{118}}$

$\therefore$ Taxable value for toy car is ₹ $\boxed{2000}$

(ii) Solve the following subquestions (any two) :

i. $5m^2 = 22m + 15$

$\therefore 5m^2 - 22m - 15 = 0$

$\therefore 5m^2 - 25m + 3m - 15 = 0$

$\therefore 5m(m-5) + 3(m-5) = 0$

$\therefore (m-5)(5m+3) = 0$

$5 \times -15 = -75$

$-25 \quad +3$

$-25 \times 3 = -75$

$-25 + 3 = -22$

By using the property, if the product of two numbers is zero; then at least one of them is zero, we get

$m - 5 = 0$ or $5m + 3 = 0$

$\therefore m = 5$ or $5m = -3$

$\therefore m = 5$ or $m = \frac{-3}{5}$

$\therefore$ The roots of the given quadratic equation are 5 and $-\frac{3}{5}$.

ii. Let the digit in unit's place be x and that in the ten's place be y.

$\therefore$ the original number $= 10y + x$

The number obtained by interchanging the digits is $10x + y$

According to the first condition, two digit number + the number obtained by interchanging the digits $= 143$

$\therefore 10y + x + 10x + y = 143$

$\therefore 11x + 11y = 143$

$\therefore x + y = 13$ ...(I) [Dividing both sides by 11]

According to the second condition, digit in unit's place $=$ digit in the ten's place +3

$\therefore x = y + 3$

$\therefore x - y = 3$ ...(ii)

Adding equations (i) and (ii), we get

$$\begin{array}{rl} & x + y = 13 \\ + & x - y = 3 \\ \hline & 2x \quad = 16 \end{array}$$

$\therefore x = 8$

Substituting $x = 8$ in equation (i), we get

$x + y = 13$

$\therefore 8 + y = 13$

$\therefore y = 5$

The original number is $10y + x = 10(5) + 8$

$= 50 + 8$

$= 58$

iii. $FV = ₹10, MV = ₹25,$ Number of shares $= 50$

i. Sum invested $=$ Number of shares $\times MV$

$= 50 \times 25 = ₹1250$

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ii. Dividend per share = 30 $\%of FV$

$= \frac{30}{100} \times 10 = ₹3$

$\therefore$ Total dividend received $= 50 \times 3 = ₹150$

iii. Rate of return $= \frac{\text{Total dividend}}{\text{Sum invested}} \times 100$

$= \frac{150}{1250} \times 100 = 12\%$

iv. S is the sample space

S = {1, 2, 3, 4, 5, 6}

$\therefore$ n(S) = $\boxed{6}$

a. Event A : Prime number on the upper face

$A = \{2, 3, 5\}$

$\therefore n(A) = \boxed{3}$

$P(A) = \frac{n(A)}{n(S)}$

$\therefore P(A) = \frac{3}{\boxed{6}} = \boxed{\frac{1}{2}}$

b. Event B : Even number on the upper face

$B = \{2, 4, 6\}$

$\therefore n(B) = \boxed{3}$

$P(B) = \frac{n(B)}{n(S)}$

$\therefore P(B) = \boxed{\frac{3}{6}} = \frac{1}{2}$

4. Solve the following subquestions (any two) :

(i) Let the time taken by bigger tap be 'x' minutes

$\therefore$ Smaller tap takes $(x + 3)$ minutes.

Part of tank filled by bigger tap in 1 minute $= \frac{1}{x}$

Part of tank filled by smaller tap in 1 minute $= \frac{1}{x+3}$

Total time taken by both the taps to fill the tank

$= 3\frac{1}{13}$ minutes

$= \frac{40}{13}$ minutes

Part of tank filled by both the taps in 1 minute $= \frac{13}{40}$

$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\therefore \frac{x+3+x}{x^2+3x} = \frac{13}{40}$

$\therefore (2x + 3)40 = 13\left(x^2 + 3x\right)$

$\therefore 80x + 120 = 13x^2 + 39x$

$\therefore 13x^2 + 39x - 80x - 120 = 0$

$\therefore 13x^2 - 41x - 120 = 0$

$\therefore 13x^2 - 65x + 24x - 120 = 0$

$\therefore 13x(x - 5) + 24(x - 5) = 0$

$\therefore (x - 5)(13x + 24) = 0$

13×-120

$= -1560$

$-65 \quad 24$

$-65 \times 24 = -1560$

$-65 + 24 = -41$

$\therefore (x - 5) = 0$ or $(13x + 24) = 0$

$\therefore x = 5$ or $x = \frac{-24}{13}$

Time taken cannot be negative.

$\therefore x = 5$ and $x + 3 = 5 + 3 = 8$ minutes

$\therefore$ Time taken by bigger tap is 5 minutes and smaller tap is 8 minutes.

(ii) Total number of students $= 180$

$\therefore 25 + x + 30 + 2x + 65 = 180$

$\therefore 3x + 120 = 180$

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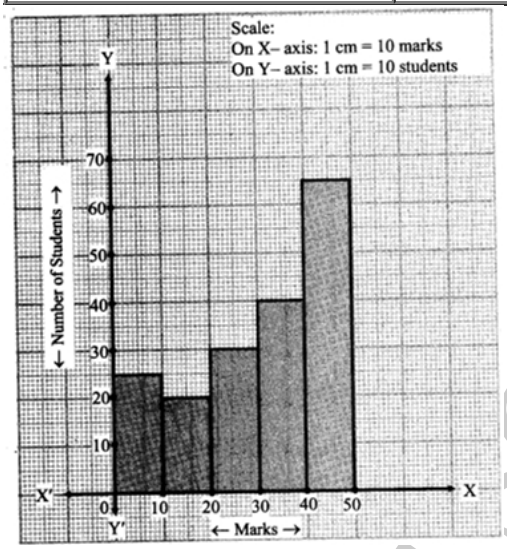
$\therefore 3x = 180 - 120$

$\therefore 3x = 60$

$\therefore x = \frac{60}{3}$

$\therefore \boldsymbol{x} = 20$

Marks	Number of Students
$0 - 10$	25
$10 - 20$	$x = 20$
$20 - 30$	30
$30 - 40$	$2x = 2 \times 20 = 40$
$40 - 50$	65



(iii) For an A.P., let a be the first term and d be the common difference.

The sum of first n terms of an A.P. is given by

$S_n = \frac{n}{2}[2a + (n-1)d]$

According to the given condition,

$S_p = S_q$

$\therefore \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\therefore p[2a + (p-1)d] = q[2a + (q-1)d]$

$\therefore 2ap + p(p-1)d = 2aq + q(q-1)d$

$\therefore 2ap + p^2d - pd = 2aq + q^2d - qd$

$\therefore 2ap + p^2d - pd - 2aq - q^2d + qd = 0$

$\therefore (2ap - 2aq) + \left(p^2d - q^2d\right) - (pd - qd) = 0$

$\therefore 2a(p-q) + d\left(p^2 - q^2\right) - d(p-q) = 0$

$\therefore 2a(p-q) + d(p+q)(p-q) - d(p-q) = 0$

$\therefore (p-q)[2a + d(p+q) - d] = 0$

$\therefore (p-q)[2a + (p+q-1)d] = 0$

But $p \neq q$ ...[Given]

$\therefore 2a + (p+q-1)d = 0$

Sum of first $(p+q)$ terms,

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}(0)$ ..[From (i)]

$\therefore S_{p+q} = 0$

$\therefore$ The sum of the first $(p+q)$ terms is zero.

5. Solve the following subquestions (any one) :

(i)

Age group (in years)	No. of persons	Measurement of central angle
$20 - 25$	80	

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		$\frac{\boxed{80}}{200} \times 360 = \boxed{144^0}$
$25-30$	60	$\frac{60}{200} \times 360 = \boxed{108^\circ}$
$30-35$	35	$\frac{35}{200} \times \boxed{360} = 63^\circ$
$35-40$	25	$\frac{25}{200} \times 360 = \boxed{45^\circ}$
Total	**200**	360°

(ii) The given simultaneous equations are

$3x - 2y = \frac{5}{2}$

$\therefore 6x - 4y = 5$ ...(i) [Multiplying both sides by 2]

$\frac{1}{3}x + 3y = \frac{-4}{3}$

$\therefore x + 9y = -4$ ...(ii) [Multiplying both sides by 3]

Equations (i) and (ii) are in $ax + by = c$ form.

Comparing the given equations with $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, we get $a_1 = 6, b_1 = -4, c_1 = 5$ and $a_2 = 1, b_2 = 9, c_2 = -4$

$\therefore D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 6 & -4 \\ 1 & 9 \end{vmatrix} = (6 \times 9) - (-4 \times 1)$

$= 54 - (-4)$

= 54 + 4 = 58 $\neq$ 0

$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ -4 & 9 \end{vmatrix} = (5 \times 9) - (-4 \times -4)$

= 45 - 16 = 29

$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 1 & -4 \end{vmatrix} = (6 \times -4) - (5 \times 1)$

$= -24 - 5 = -29$

$\therefore$ By Cramer's rule, we get

$x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$

$\therefore x = \frac{29}{58}$ and $y = \frac{-29}{58}$

$\therefore x = \frac{1}{2}$ and $y = \frac{-1}{2}$

$\therefore (x, y) = \left(\frac{1}{2}, \frac{-1}{2}\right)$ is the solution of the given simultaneous equations.

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