Solution

ALGEBRA

Class 10 - Mathematics - I

1. (i) Choose the correct alternative from given :

   i. **(d)** $x^2 + 4x = 11 + x^2$
   Explanation:
   $x^2 + 4x = 11 + x^2$
   $\therefore x^2 - x^2 + 4x = 11$
   $\therefore 4x = 11$ ... (i)
   Equation (i) is a linear equation as degree of equation is 1.
   $\therefore x^2 + 4x = 11 + x^2$ is not a quadratic equation.

   ii. **(d)** 2
   Explanation:
   Putting $y = 1$ in $x + 2y = 4$, we get
   $x + 2(1) = 4$
   $\therefore x + 2 = 4$
   $\therefore x = 2$

   iii. **(c)** 55
   Explanation:
   First 10 natural numbers are $1, 2, 3, \ldots, 9, 10$.
   The above sequence is an A.P.
   $\therefore t_1 = 1, t_{10} = 10$
   $\therefore S_n = \frac{n}{2}(t_1 + t_{10}) = \frac{10}{2}(1 + 10)$
   $= 5(11)$
   $= 55$

   iv. **(d)** CGST
   Explanation:
   CGST

   (ii)
   i.
   $$\begin{array}{r} 17x + 15y = 11 \\ 15x + 17y = 21 \\ - \quad - \quad - \\ \hline 2x - 2y = -10 \end{array}$$
   $\therefore x - y = -5$ ...[Dividing both sides by 2]

   ii. $a = t_1 = 6, d = -3$
   $\therefore t_2 = t_1 + d = 6 - 3 = 3$
   $t_3 = t_2 + d = 3 - 3 = 0$

   iii. Brokerage paid on one share $= 2\%$ of MV
   $= \frac{2}{100} \times 150$
   $= ₹3$

   iv. **(a)** $\frac{1}{6}$
   Explanation:
   Sample space $(S) = \{1, 2, 3, 4, 5, 6\}$
   $n(S) = 6$
   Let A be the event that the number appearing on upper face of the die is less than 2.
   $\therefore A = \{1\}$
   $\therefore n(A) = 1$
   $\therefore P(A) = \frac{n(A)}{n(S)}$
   $\therefore P(A) = \frac{1}{6}$

2. (i) Complete the following activities and rewrite it (any two) :

i. First term $= a = 6$, common difference $= d = 3$, $S_{27} =$?

$S_n = \frac{n}{2}[\boxed{2a} + (n-1)d]$

$\therefore S_{27} = \frac{27}{2}[12 + (27-1)\boxed{3}]$

$= \frac{27}{2} \times \boxed{90} = 27 \times 45$

$\therefore S_{27} = \boxed{1215}$

ii. $x^2 + 8x + 15 = 0$

$\therefore x^2 + 5x + 3x + 15 = 0$

$\therefore x(x+5) + 3(x+5) = 0$

$\therefore (x+5)(x+3) = 0$

15

+5 +3

+5 × +3 = +15

+5 + 3 = +8

By using the property, if the product of two numbers is zero, then at least one of them is zero, we get

$x + 5 = 0$ or $x + 3 = 0$

$\therefore x = -5$ or $x = -3$

$\therefore$ The roots of the given quadratic equation are -5 and -3.

iii. Total number of pens in the box

$= 5 + 8 + 3 = 16$

$\therefore n(S) = 16$

Let A be the event that Rutuja picks a blue pen.

Total number of blue pens = 8

$\therefore n(A) = 8$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{8}{16}$

$\therefore P(A) = \frac{1}{2}$

$\therefore$ The probability that Rutuja picks a blue pen is $\frac{1}{2}$.

(ii) Solve the following subquestions (any four) :

i. $x + y = 6$ ...(i)

$x - y = 4$ ...(ii)

Adding equations (i) and (ii), we get

$$\begin{array}{rl} & x + y = 6 \\ + & x - y = 4 \\ \hline & 2x \quad = 10 \end{array}$$

$\therefore x = \frac{10}{2} = 5$

Substituting $x = 5$ in equation (i), we get

$5 + y = 6$

$\therefore y = 6 - 5 = 1$

$\therefore (x, y) = (5, 1)$ is the solution of the given simultaneous equations.

ii. Let $\alpha = -3$ and $\beta = -7$

$\therefore \alpha + \beta = -3 - 7 = -10$

and $\alpha\beta = (-3)(-7) = 21$

$\therefore$ The required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (-10)x + 21 = 0$

$\therefore x^2 + 10x + 21 = 0$

iii. $a = 10, d = 5$ ...[Given]

$\therefore t_1 = a = 10$

$t_2 = t_1 + d = 10 + 5 = 15$

$t_3 = t_2 + d = 15 + 5 = 20$

$t_4 = t_3 + d = 20 + 5 = 25$

$\therefore$ The required A.P. is $10, 15, 20, 25, \ldots$

iv. Suppose S is sample space.

$\therefore n(S)=52$

Event A: Card drawn is a red card.

$\therefore$ Total red cards = $\boxed{13}$ hearts + 13 diamonds

$\therefore n(A)=\boxed{26}$

$P(A)=\frac{\boxed{n(A)}}{n(S)}$

$\therefore P(A)=\frac{26}{52}$

$\therefore P(A)=\boxed{\frac{1}{2}}$

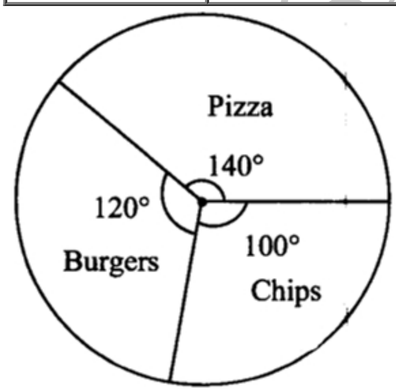
v.

Class (Number of hours daily)	**Frequency (Number of workers)**	**Cumulative frequency (less than)**
$8-10$	150	150
$10-12$	500	$150+500=650$
$12-14$	300	$650+300=950$
$14-16$	50	$950+50=1000$
Total	$N=1000$	-

3. (i) Complete the following activity and rewrite it (any one) :

i. Measure of central angle $(\theta)=\frac{\text{Number of scores in the components}}{\text{Total number of scores}}\times 360^{\circ}$

Item	**No. of people**	**Measure of central angle (θ)**
Pizza	70	$\frac{70}{180}\times 360^{\circ}=140^{\circ}$
Burgers	60	$\frac{60}{180}\times 360^{\circ}=120^{\circ}$
Chips	50	$\frac{50}{180}\times 360^{\circ}=100^{\circ}$
Total	180	360°



ii. Printed price of dress $=₹2000$,

Rate of discount $=5\%$

Amount of discount $=5\%$ of Printed price

$=\frac{5}{100}\times 2000$

$=₹100$

Taxable value $=$ Printed price - Discount

$=2000-100$

$=₹1900$

Rate of GST $=5\%$

$\therefore$ GST $=5\%$ of taxable value

$=\frac{5}{100}\times 1900$

$\therefore$ GST $=$ ₹ 95

$\therefore$ Purchase price of the dress $=$ Taxable value + GST

$= 1900 + 95$

$= ₹1995$

$\therefore$ The purchase price of the dress for the customer is ₹ 1995.

(ii) Solve the following subquestions (any two) :

i. $5m^2 + 13m + 8 = 0$

Comparing the above equation with

$am^2 + bm + c = 0$, we get

$a = 5,\ b = 13, c = 8$

$b^2 - 4ac = (13)^2 - 4 \times 5 \times 8$

$= 169 - 160 = 9$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-13 \pm \sqrt{9}}{2(5)}$

$= \frac{-13 \pm 3}{10}$

$\therefore\ m = \frac{-13+3}{10}$ or $m = \frac{-13-3}{10}$

$\therefore\ m = \frac{-10}{10}$ or $m = \frac{-16}{10}$

$\therefore\ m = -1$ or $m = \frac{-8}{5}$

$\therefore$ The roots of the given quadratic equation are -1 and $\frac{-8}{5}$.

ii. $(3, -1)$ is the point of intersection of the lines $ax + by = 9$ and $bx + ay = 5$

$\therefore$ Point $(x, y) = (3, -1)$ satisfies the two equations.

$ax + by = 9$ ...(i)

$bx + ay = 5$ ...(ii)

$\therefore$ Putting $x = 3$ and $y = -1$ in the above equations, we get

$3a - b = 9$ ...(iii)

$3b - a = 5$ ...(iv)

Multiplying equation (iv) by 3, we get

$-3a + 9b = 15$ ...(v)

Adding equations (iii) and (v), we get

$$\begin{array}{r} 3a - b = 9 \\ -3a + 9b = 15 \\ \hline 8b = 24 \end{array}$$

$\therefore b = \frac{24}{8}$

$\therefore b = 3$

Putting $b = 3$ in equation (iii), we get

$3a - b = 9$

$\therefore 3a - 3 = 9$

$\therefore 3a = 9 + 3$

$\therefore 3a = 12$

$\therefore a = 4$

$\therefore a = 4$ and $b = 3$

iii. Taxable value of 1 tin $= ₹2,800$

$\therefore$ Taxable value of 2 tins $= 2 \times 2,800$

= ₹ $5,600$

Rate of GST $= 28\%$

$\therefore$ Rate of CGST $=$ Rate of SGST $= 14\%$

CGST $= 14\%$ of taxable value

$= \frac{14}{100} \times 5,600$

$\therefore$ CGST $= ₹784$

$\therefore$ SGST $= CGST =$ ₹ 784

$\therefore$ The amount of CGST and SGST charged in the tax invoice is $₹784$ each.

iv. Let the 3 red balls be R_1, R_2, R_3,

3 black balls be B_1, B_2, B_3,

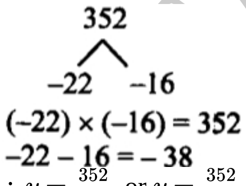
3 white balls be W_1, W_2, W_3,
3 green balls be G_1, G_2, G_3
$\therefore$ Sample space,
$S = \{R_1, R_2, R_3,\ B_1,\ B_2,\ B_3,\ W_1,\ W_2,\ W_3\ , G_1, G_2, G_3\}$
$\therefore n(S) = 12$

i. Let A be the event that the ball drawn is white.
$\therefore A = \{W_1,\ W_2,\ W_3\}$
$\therefore n(A) = 3$
$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$
$\therefore P(A) = \frac{1}{4}$

ii. Let B be the event that the ball drawn is not white.
$\therefore B = \{R_1, R_2, R_3,\ B_1,\ B_2,\ B_3, G_1, G_2, G_3\}$
$\therefore n(B) = 9$
$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{9}{12} = \frac{3}{4}$
$\therefore P(B) = \frac{3}{4}$

4. Solve the following subquestions (any two) :

(i) Let the two numbers be x and y.
According to the first condition,
Product of two numbers is 352
$\therefore xy = 352$
$\Rightarrow y = \frac{352}{x}$ ...(i)
According to the second condition,
Mean $= 19$
$\therefore \frac{x+y}{2} = 19$
$\therefore x + y = 38$
Substituting the value of y from equation (i), we get
$\therefore x + \frac{352}{x} = 38$
$\therefore x^2 + 352 = 38x$
$\therefore x^2 - 38x + 352 = 0$
$\therefore x^2 - 22x - 16x + 352 = 0$
$\therefore x(x - 22) - 16(x - 22) = 0$
$\therefore (x - 16)(x - 22) = 0$
$\therefore x - 16 = 0$ or $x - 22 = 0$
$\therefore x = 16$ or $x = 22$

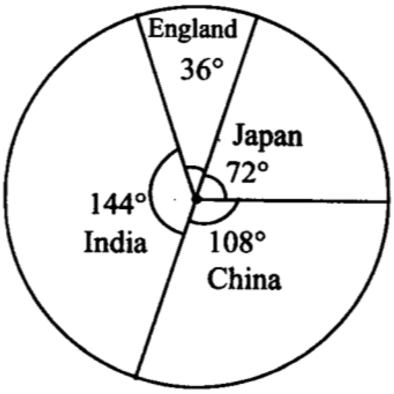


$\therefore y = \frac{352}{16}$ or $y = \frac{352}{22} \ldots [$ From (i)]
$\therefore y = 22$ or $y = 16$
$\therefore$ The required numbers are 16, 22 or 22, 16.

(ii) Total percentage of world population $= 100$
Measure of central angle $(\theta) = \frac{\text{Percentage of components}}{100} \times 360^\circ$

Country	**Percentage of total population**	**Measure of central angle (θ)**
Japan	20	$\frac{20}{100} \times 360^\circ = 72^\circ$
England	10	$\frac{10}{100} \times 360^\circ = 36^\circ$
India	40	$\frac{40}{100} \times 360^\circ = 144^\circ$
China	30	$\frac{30}{100} \times 360^\circ = 108^\circ$

Total	100	360°



(iii)The instalments are in A. P.

Amount repaid in 10 instalments (S_{10})

$=$ Amount borrowed + total interest

$\therefore S_{10} = 4000 + 500 = 4500$

Number of instalments (n) $= 10$

Each instalment is less than the preceding instalment by ₹ 10.

$\therefore d = -10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{10} = \frac{10}{2}[2a + (10-1)(-10)]$

$\therefore 4500 = 5[2a + 9(-10)]$

$\therefore \frac{4500}{5} = 2a - 90$

$\therefore 900 = 2a - 90$

$\therefore 2a = 900 + 90$

$\therefore 2a = 990$

$\therefore a = \frac{990}{2}$

$\therefore a = 495$

Now, $t_n = a + (n-1)d$

$\therefore t_{10} = 495 + (10-1)(-10)$

$\therefore t_{10} = 495 + 9(-10)$

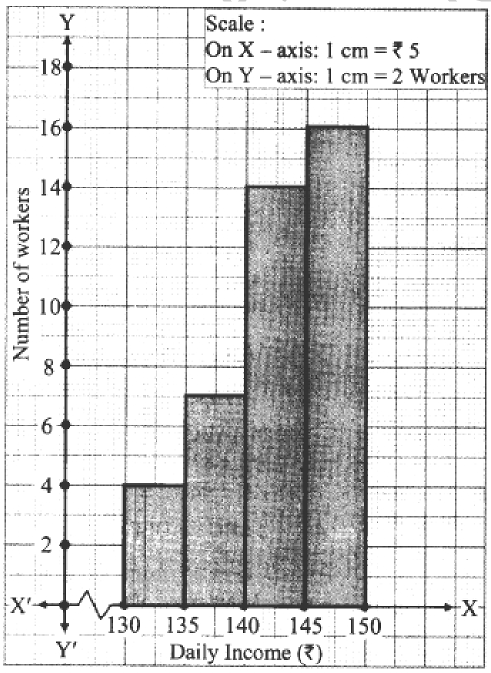
$\therefore t_{10} = 495 - 90$

$\therefore t_{10} = 405$

$\therefore$ Amount of the first instalment is 495 and that of the last instalment is 405.

5. Solve the following subquestions (any one) :

(i)



(ii) The given simultaneous equations are

$3x - 4y = 10$ ...(i)

$4x + 3y = 5$ ...(ii)

Equations (i) and (ii) are in $ax + by = c$ form.

Comparing the given equations with $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, we get $a_1 = 3, b_1 = -4, c_1 = 10$ and $a_2 = 4, b_2 = 3, c_2 = 5$

$\therefore D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & -4 \\ 4 & 3 \end{vmatrix} = (3 \times 3) - (-4 \times 4)$

$= 9 - (-16)$

$= 9 + 16 = 25 \neq 0$

$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = \begin{vmatrix} 10 & -4 \\ 5 & 3 \end{vmatrix} = (10 \times 3) - (-4 \times 5)$

$= 30 - (-20)$

$= 30 + 20 = 50$

$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & 10 \\ 4 & 5 \end{vmatrix} = (3 \times 5) - (10 \times 4)$

$= 15 - 40 = -25$

$\therefore$ By Cramer's rule, we get

$x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$

$\therefore x = \frac{50}{25}$ and $y = \frac{-25}{25}$

$\therefore x = 2$ and $y = -1$

$\therefore (x, y) = (2, -1)$ is the solution of the given simultaneous equations.