SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Let A be a skew-symmetric matrix of order 3. If |A| = x, then $(2023)^x$ is equal to: **[1]**

   a) 2023 b) $(2023)^2$

   c) $\frac{1}{2023}$ d) 1

2. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is **[1]**

   a) 1 b) 0

   c) -1 d) 3

3. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is **[1]**

   a) 10 b) -7

   c) 7 d) 8

4. If A is skew symmetric matrix of order 3, then the value of |A| is: **[1]**

   a) 9 b) 3

   c) 0 d) 27

5. A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are **[1]**

a) $\frac{1}{3}, \frac{-1}{3}, \frac{2}{3}$ b) $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

c) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ d) $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$

6. Degree of the differential equation sin x + $\cos\left(\frac{dy}{dx}\right) = y^2$ is **[1]**

a) 2 b) not defined

c) 0 d) 1

7. The common region determined by all the constraints of a linear programming problem is called: **[1]**

a) an optimal region b) a feasible region

c) a bounded region d) an unbounded region

8. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is **[1]**

a) 5 b) 12

c) 7 d) 1

9. $\int 4^x 3^x$ dx equals **[1]**

a) $\left(\frac{4^x \cdot 3^x}{\log 4 \cdot \log 3}\right) + C$ b) $\frac{3^x}{\log 3} + C$

c) $\frac{12^x}{\log 12} + C$ d) $\frac{4^x}{\log 4} + C$

10. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to: **[1]**

a) I b) I - A

c) I + A d) 0

11. A Linear Programming Problem is as follows: **[1]**

Maximize/Minimize objective function Z = 2x - y + 5

Subject to the constraints

$3x + 4y \leq 60$

$x + 3y \leq 30$

$x \leq 0, y \geq 0$

In the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0,0), then which of the following is true?

a) Minimum value of Z is -5 b) At two corner points, value of Z are equal

c) Maximum value of Z is 40 d) Difference of maximum and minimum values of Z is 35

12. The two lines x = ay + b, z = cy + d; and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if: **[1]**

a) $aa' + cc' = -1$ b) aa' + cc' = 1

c) $\frac{a}{a'} + \frac{c}{c'} = -1$ d) $\frac{a}{a'} + \frac{c}{c'} = 1$

13. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj(adj A)) is **[1]**

a) 14^3

b) 14

c) 14^4

d) 14^2

14. If $P\left(\frac{A}{B}\right)$ = 0.3, P(A) = 0.4 and P(B) = 0.8, then $P\left(\frac{B}{A}\right)$ is equal to: **[1]**

a) 0.4

b) 0.3

c) 0.6

d) 0.06

15. If x = a cos θ + b sin θ, y = a sin θ - b cos θ, then which one of the following is true? **[1]**

a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

c) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

d) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

16. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a}\cdot\hat{b}$ is equal to: **[1]**

a) $\pm\frac{3}{4}$

b) $\pm\frac{3}{5}$

c) $\pm\frac{4}{3}$

d) $\pm\frac{4}{5}$

17. Let f(x) $= \begin{cases} \frac{x^2-2x-3}{x+1^2}, \text{ when } x \neq -1 \\ k, \text{ when } x = -1 \end{cases}$ If f(x) is continuous at x = -1 then k = **[1]**

a) 2

b) 4

c) -4

d) -3

18. If α, β, γ are the angles that a line makes with the positive direction of x, y, z axis, respectively, then the direction cosines of the line are **[1]**

a) tan α, tan β, tan γ

b) sin α, sin β, sin γ

c) cos α, cos β, cos γ

d) $\cos^2\alpha$, $\cos^2\beta$, $\cos^2\gamma$

19. **Assertion (A):** A particle moving in a straight line covers a distance of x cm in t second, where $x = t^3 + 3t^2 - 6t + 18$. The velocity of particle at the end of 3 seconds is 39 cm/s. **[1]**

Reason (R): Velocity of the particle at the end of 3 seconds is $\frac{dx}{dt}$ at t = 3.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A)** Let f : R $\rightarrow$ R be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 are ± 4. **[1]**

Reason (R): A function f : A $\rightarrow$ B is called a one-one function, if distinct elements of A have distinct images in B.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. **[2]**

OR

Find the principal value of $\tan^{-1}(\sqrt{3})$.

22. Find the interval in function $-2x^3 - 9x^2 - 12x + 1$ is increasing or decreasing: **[2]**

23. Find the intervals of function $f(x) = 2x^3 - 24x + 5$ is **[2]**

a. increasing

b. decreasing.

OR

A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

24. Find the integral: $\int \frac{dx}{x^2-6x+13}$ **[2]**

25. Find the intervals of function $f(x) = (x - 1)(x - 2)^2$ is **[2]**

a. increasing

b. decreasing.

Section C

26. Evaluate: $\int \frac{x^3}{(x-1)(x^2+1)}dx$ **[3]**

27. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red? **[3]**

28. Find $\int \frac{2x}{(x^2+1)(x^4+4)}dx$. **[3]**

OR

Find: $\int \frac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi}d\phi$

29. Solve the initial value problem: $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0, y(1) = \frac{\pi}{2}$ **[3]**

OR

Verify that $y = e^x \cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

30. Solve the Linear Programming Problem graphically: **[3]**

Minimize Z = 30x + 20y Subject to

$x + y \leq 8$

$x + 4y \geq 12$

$5x + 8y = 20$

$x, y \geq 0$

OR

Solve the following linear programming problem graphically:

Maximise: Z = x + 2y

subject to the constraints:

$x + 2y \geq 100$,

$2x - y \leq 0$,

$2x + y \leq 200$,

$x \geq 0, y \geq 0$.

31. Show that the function $f(x) = |x - 3|$, $x \in R$, is continuous but not differentiable at x = 3. **[3]**

Section D

32. Sketch the graph of y = Ix + 3I and evaluate the area under the curve y = Ix + 3I above X - axis and between x = **[5]**

- 6 to x = 0.

33. Given, A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following: **[5]**

a. an injective mapping from A to B

b. a mapping from A to B which is not injective

c. a mapping from B to A.

OR

Let R be relation defined on the set of natural number N as follows:

R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

34. Using matrices, solve the following system of equations. **[5]**

x-y+ 2z = 7

3x + 4y - 5z = - 5

and 2x- y + 3z = 12

35. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and 3x - 2y + z + 5= 0 = 2x + 3y + 4z - 4 intersect. Find the equation of the plane in which they lie and also their point of intersection. **[5]**

OR

Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

i. Find the probability that daughter is at one end, given that father and mother are in the middle. (1)

ii. Find the probability that mother is at right end, given that son and daughter are together. (1)

iii. Find the probability that father and mother are in the middle, given that son is at right end. (2)

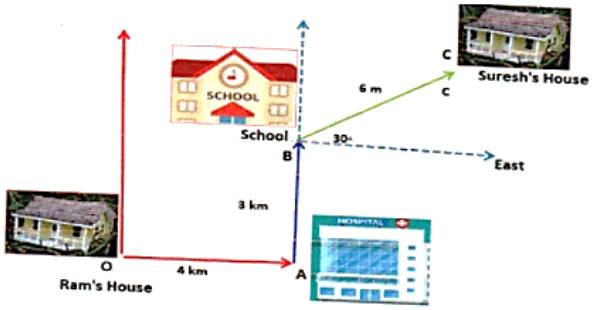
OR

Find the probability that father and son are standing together, given that mother and daughter are standing together. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at

point B school is situated. Suresh's house is at 30^o east, 6 km from point B.



i. Find the displacement between Ram's house and school. (1)

ii. How many km Ram travels to reach school? (1)

iii. What is the vector distance from school to Suresh's home? (2)

OR

What is the displacement from Ram's house to Suresh house? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

i. Express cost of tank as a function of height(h). (1)

ii. Verify by second derivative test that cost is minimum at critical point. (1)

iii. Find the value of h at which c(h) is minimum. (2)

OR

Find the minimum cost of tank? (2)