Solution

MATHEMATICS

Class 12 - Mathematics

Section A

1.
(d) 1
Explanation:
1

2.
(c) -1
Explanation:
-1

3.
(b) -7
Explanation:
-7
$\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$
=6(3-4) -0(6-4)+(-1)(2-1)
= 6 (-1) +0+ (-1)
= -6 -1
=-7

4.
(c) 0
Explanation:
Determinant value of skew-symmetric matrix is always '0'.

5.
(c) $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$
Explanation:
If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.
$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$
$\overrightarrow{\mathrm{b}} = 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$
$\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$
$= 2\hat{\imath} - \hat{j} + 2\hat{k}$
So,
$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 1 + 2^2}$ =3

So, Director cosine
$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

6.
(b) not defined
Explanation:
not defined

7.

(b) a feasible region

Explanation:

a feasible region

8.

(c) 7

Explanation:

7

9.

(c) $\frac{12^x}{\log 12} + C$

Explanation:

$\int 4^x 3^x dx$
$= \int (12)^x dx$
$= \frac{12^x}{\log 12} + c$

10. **(a)** I

Explanation:

Given that $A^2 = A$

Calculating value of $(I - A)^3 + A$:

$(I - A)^3 + A = I^3 - 3I^2A + 3IA^2 - A^3 + A$

$= I - A^2A - 3A + 3A^2 + A$ ($\because I^n = I$ and $IA = A$)

$= I - AA - 3A + 3A + A$ ($\because A^2 = A$)

$= I + A^2 - 3A + 3A + A$
$= I$

Hence, $(I - A)^3 + A = I$

11. **(a)** Minimum value of Z is -5

Explanation:

Corner points	Value of Z = 2x - y + 5
A(0, 10)	Z = 2(0) - 10 + 5 = -5 (Minimum)
B(12, 6)	Z = 2(12) - 6 + 5 = 23
C(20, 0)	Z = 2(20) - 0 + 5 = 45 (Maximum)
O(0, 0)	Z = 0(0) - 0 + 5 = 5

So the minimum value of Z is -5.

12. **(a)** $aa' + cc' = -1$

Explanation:

x = ay + b, z = cy + d
$L_1 : \frac{x-b}{a} = y = \frac{z-d}{c}$
x = $a'y + b', z = c'y + d'$
$L_2 : \frac{x-b'}{a'} = y = \frac{z-d'}{c'}$
if two lines are perpendicular, angle between their direction ratio is $\frac{\pi}{2}$
$\cos \frac{\pi}{2} = 0$
$aa' + cc' \pm -1 = 0$
$aa' + cc' = -1$

13.

(c) 14^4

Explanation:

$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

|A| = 14 det(adjA)= $det(A)^{3-1}$ = $det(A)^2$.Here the operation is done two times.so,

det (adj(adj A)) = $|A|^{(n-1)^2}$

det (adj(adj A)) = $14^{(3-1)^2} = 14^4$

14.

(c) 0.6

Explanation:

0.6

15. **(a)** $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Explanation:

$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

16.

(d) $\pm\frac{4}{5}$

Explanation:

$\pm\frac{4}{5}$

17.

(c) -4

Explanation:

We have,

$\Rightarrow f(x) = \frac{x^2-2x-3}{x+1}$ is continuous at x = 0.

$\Rightarrow f(x) = \lim_{x\to-1} \frac{(x+1)(x-3)}{x+1}$

$\Rightarrow f(x) = \lim_{x\to-1} x - 3$

$\Rightarrow$ f(x) = -4

$\therefore$ k = -4

18.

(c) cos α, cos β, cos γ

Explanation:

cos α, cos β, cos γ

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

We have,

x = $t^3 + 3t^2$ - 6t + 18

Velocity, $v = \frac{dx}{dt} = 3t^2 + 6t - 6$

Thus, velocity of the particle at the end of 3 seconds is

$\left(\frac{dx}{dt}\right)_{t=3} = 3(3)^2 + 6(3) - 6$

= 27 + 18 - 6 = 39 cm/s

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion Consider $x^2 + 1 = 17$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

Hence, pre-images of 17 are ± 4.

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Section B

21. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$.

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right) \begin{bmatrix} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ and \tan^{-1}(-x) = -\tan^{-1} x \end{bmatrix}$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$

$= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$

OR

We know that the range of the principal-value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Let $\tan^{-1}(\sqrt{3}) = \theta$, Then, we have,

$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of $\tan^{-1}(\sqrt{3})$ is equal to $\frac{\pi}{3}$

22. It is given that function $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12$

$\Rightarrow f'(x) = -6(x^2 + 3x + 2)$

$\Rightarrow f'(x) = -6(x + 1)(x + 2)$

If $f'(x) = 0$, then we get,

$\Rightarrow x = -1$ and -2

So, the points $x = -1$ and $x = -2$ divide the real line into three disjoint intervals, $(-\infty, -2), (-2, -1)$ and $(-1, \infty)$

So, in intervals $(-\infty, -2), (-1, \infty)$

$f'(x) = -6(x + 1)(x + 2) < 0$

Therefore, the given function 'f ' is strictly decreasing for $x < -2$ and $x > -1$

Further, in interval $(-2, -1)$

$f'(x) = -6(x + 1)(x + 2) > 0$

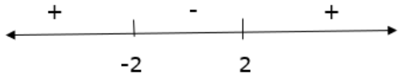
Therefore, the given function (f) is strictly increasing for $-2 < x < -1$

23. Given function is $f(x) = 2x^3 - 24x + 5$

$f'(x) = 6x^2 - 24$

$f'(x) = 6(x^2 - 4)$

$= 6(x - 2)(x + 2)$



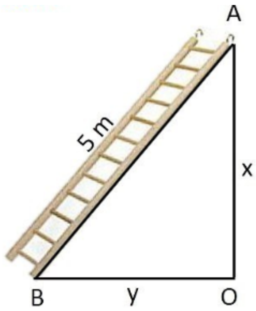
Function f(x) is decreasing for $x \in [-2, 2]$ and increasing in $x \in (-\infty, -2) \cup (2, \infty)$.

OR

Let AB be the ladder & length of ladder is 5m

i..e, AB = 5

& OB be the wall & OA be the ground.



Suppose OA = x & OB = y

Given that

The bottom of the ladder is pulled along the ground, away the wall at the rate of 2cm/s

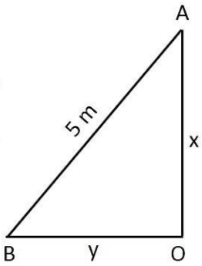
i.e., $\frac{dx}{dt}$ = 2cm/sec ..... (i)

We need to calculate at which rate height of ladder on the wall.

Decreasing when foot of the ladder is 4 m away from the wall

i.e. we need to calculate $\frac{dy}{dt}$ when x = 4 cm

Wall OB is perpendicular to the ground OA



Using Pythagoras theorem,we get

$(OB)^2 + (OA)^2 = (AB)^2$

$y^2 + x^2 = (5)^2$

$y^2 + x^2 = 24$ .... (ii)

Differentiating w.r.t. time,we get

$\frac{d(y^2+x^2)}{dt} = \frac{d(25)}{dt}$

$\frac{d(y^2)}{dt} + \frac{d(x^2)}{dt} = 0$

$\frac{d(y^2)}{dt} \times \frac{dy}{dy} + \frac{d(x^2)}{dt} \times \frac{dx}{dx} = 0$

$2y \times \frac{dy}{dt} + 2x \times \frac{dx}{dt} = 0$

$2y \times \frac{dy}{dx} + 2x \times (2) = 0$

$2y\frac{dy}{dt} + 4x = 0$

$2y\frac{dy}{dt} = -4x$

$\frac{dy}{dt} = \frac{-4x}{2y}$

We need to find $\frac{dy}{dt}$ when x = 4cm

$\frac{dy}{dt}\Big|_{x=4} = \frac{-4\times4}{2y}$

$\frac{dy}{dt}\Big|_{x=4} = \frac{-16}{2y}$ ..... (iii)

Finding value of y

From (ii)

$x^2 + y^2 = 25$

Putting x = 4

$(4)^2 + y^2 = 25$

$y^2 = 9$

y = 3

24. We have $x^2 - 6x + 13 = x^2 - 6x + 3^2 - 3^2 + 13 = (x - 3)^2 + 4$

So, $\int \frac{dx}{x^2-6x+13} = \int \frac{1}{(x-3)^2+2^2}dx$

Let x – 3 = t ⇒ dx = dt

Therefore,

$\int \frac{dx}{x^2-6x+13} = \int \frac{dt}{t^2+2^2} = \frac{1}{2}\tan^{-1}\frac{t}{2} + \text{C}$

$= \frac{1}{2}\tan^{-1}\frac{x-3}{2} + C$

25. Given function is f(x) = (x - 1)(x - 2)^2 = x^2 - 4x + 4 (x - 1)

= x^3 - 4x^2 + 4x - x^2 + 4x - 4

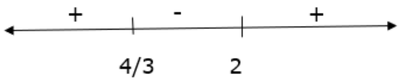
f(x) = x^3 - 5x^2 + 8x - 4

f'(x) = 3x^2 - 10x + 8

f'(x) = 3x^2 - 6x - 4x + 8

= 3x(x - 2) -4(x - 2)

= (3x - 4)(x - 2)



Function f(x) is decreasing for x∈[4/3, 2] and increasing in x∈$(-\infty, 4/3) \cup (2, \infty)$.

Section C

26. Let $I = \int \frac{x^3}{(x-1)(x^2+1)}dx$

$= \int \frac{(x^3-1)+1}{(x-1)(x^2+1)}dx$

$= \int \frac{x^3-1}{(x-1)(x^2+1)}dx + \int \frac{1}{(x-1)(x^2+1)}dx$

$= \int \frac{(x-1)(x^2+x+1)}{(x-1)(x^2+1)}dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \int \frac{x^2+x+1}{x^2+1}dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \int dx + \frac{1}{2}\int \frac{2x}{x^2+1}dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= x + \frac{1}{2}$ ln (x^2 + 1) + I,

Where $I = \int \frac{dx}{(x-1)(x^2+1)}$

Using partial fractions

$\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$ ..... (i)

⇒ A(x^2 + 1) + (Bx + C) (x - 1) = 1

At x = 1, $A = \frac{1}{2}$

At x = 1, $C = -\frac{1}{2}$ and $B = -\frac{1}{2}$

using these value in (Eqn. (i), we get

$I_1 = \int \left[\frac{1/2}{(x-1)} + \frac{\frac{-1}{2}x - \frac{1}{2}}{x^2+1}\right]dx$

$= \frac{1}{2}\ln(x-1) - \frac{1}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{dx}{x^2+1}$

$= \frac{1}{2}\ln(x-1) - \frac{1}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\tan^{-1}x$

$= \frac{1}{2}\ln(x-1) - \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x$

$\therefore I = x + \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\ln(x-1) - \frac{1}{4}\ln(x^2+1)\frac{-1}{2}\tan^{-1}x + c$

27. **Case (i) :** S$_1$ = {5 red balls, 5 black balls}

$\Rightarrow n(S_1) = 10$

Let us draw a red balls first, i.e., A$_1$ = {5 red balls}

$\Rightarrow n(A_1) = 5$

$P(A_1) = \frac{n(A_1)}{n(S_1)} = \frac{5}{10} = \frac{1}{2}$

Now after adding 2 balls of the same colour, i.e., when the first draw gives a red ball, two additional red balls are put in the urn so that its contents are 7 (5 + 2) red and 5 balck balls. When the first draw gives a black ball, two additional black balls are put in the urn so that its contents are 5 red and 7 (5 + 2) black balls.

Total balls = S$_2$ = {7 red balls, 5 black balls}

$\Rightarrow n(S_2) = 12$

Let us draw a red balls first, i.e., A_2 = {7 red balls}

$\Rightarrow n(A_2) = 7$

$P(A_2) = \frac{n(A_2)}{n(S_2)} = \frac{7}{12}$

$\therefore$ P (a red ball is drawn) $= \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

Case (ii) : When a black ball is drawn, i.e., A_2 = {5 red balls}

$\Rightarrow n(A_2) = 5$

$= P(A_1) = \frac{n(A_2)}{n(S_1)} = \frac{5}{12}$

Now after adding 2 balls of the same colour, i.e.,

S_2 = {5 red balls, 7 black balls}

$\Rightarrow n(S_2) = 12$

Let us draw a red balls first, i.e., A_2 = {5 red balls}

$\Rightarrow n(A_2) = 5$

$P(A_2) = \frac{n(A_2)}{n(S_2)} = \frac{5}{12}$

$\because$ P (a red ball is drawn) $= \frac{1}{2} \times \frac{5}{12} = \frac{5}{24}$

Therefore, required probability in both cases

= Probability that first ball is red and then second ball after two red are added in the urn is also red + Probability that first ball is black and second is red $= \frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}$

28. According to the question, $I = \int \frac{2x}{(x^2+1)(x^4+4)} dx$

$= \int \frac{2x}{\left[x^2+1\right]\left[(x^2)^2+4\right]} dx$

Put $x^2 = t \quad \Rightarrow \quad 2x dx = dt$

$\therefore \quad I = \int \frac{dt}{(t+1)(t^2+4)}$

Now, $\frac{1}{(t+1)(t^2+4)} = \frac{A}{t+1} + \frac{Bt+C}{t^2+4}$

$\Rightarrow 1 = A(t^2+4) + (Bt+C)(t+1)$

$\Rightarrow 1 = A(t^2+4) + (Bt^2 + Bt + Ct + C)$

$\Rightarrow 1 = t^2(A+B) + t(B+C) + (4A+C)$

Comparing the coefficients of t^2, t and constant term from both sides,

A+ B = 0 ...(i)

B+C=0 ...(ii)

4A + C = 1 ...(iii)

From Eqs. (i) and (ii), we get

A - C=0

From Eqs. (iii) and (iv), we get

$5A = 1 \quad \Rightarrow \quad A = \frac{1}{5}$

Then, $C = \frac{1}{5}, B = -\frac{1}{5}$

Now, $I = \int \frac{dt}{(t+1)(t^2+4)} = \int \frac{1}{5(t+1)} dt + \int \frac{(-1/5)t + (1/5)}{t^2+4} dt$

$= \frac{1}{5} \int \frac{dt}{t+1} - \frac{1}{5} \int \frac{t-1}{t^2+4} dt$

$= \frac{1}{5} \log|t+1| - \frac{1}{5}\left[\int \frac{t}{t^2+4} dt - \int \frac{1}{t^2+4} dt\right]$

in second integral , $let\ t^2 = u \implies 2t dt = du$

$= \frac{1}{5} \log|t+1| - \frac{1}{5}\left[\frac{1}{2}\log|u+4| - \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right] + k \left[\therefore \int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + k\right]$

$= \frac{1}{5}\log|x^2+1| - \frac{1}{10}\log|x^4+4| + \frac{1}{10}\tan^{-1}\left(\frac{x^2}{2}\right) + k$ [here K is integration constant]

OR

Let $y = \sin\phi \Rightarrow dy = \cos\phi d\phi$

Therefore, $\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi} d\phi = \int \frac{(3y-2)dy}{5-(1-y^2)-4y}$

$= \int \frac{3y-2}{y^2-4y+4} dy$

Now, we write $\frac{3y-2}{(y-2)^2} = \frac{A}{y-2} + \frac{B}{(y-2)^2}$

Therefore, 3y - 2 = A (y - 2) + B

Comparing the coefficients of y and constant term,

we get A = 3 and B = 4. for y = 2, y= 0

Therefore, the required integral is given by

$I=\int\left[\frac{3}{y-2}+\frac{4}{(y-2)^2}\right]dy$ $=3\int\frac{dy}{y-2}+4\int\frac{dy}{(y-2)^2}$

= $3\log|y-2|+4\left(-\frac{1}{y-2}\right)+C$

= $3\log|\sin\phi-2|+\frac{4}{2-\sin\phi}+C$

= $3\log(2-\sin\phi)+\frac{4}{2-\sin\phi}+C$ (since, sin ϕ $\in$[-1,1], sin ϕ< 2, 2 - sin ϕ is always positive)

29. Given that, $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$

$\Rightarrow\frac{dy}{dx}=-\frac{x-y\sin\left(\frac{y}{x}\right)}{x\sin\left(\frac{y}{x}\right)}$

This is a homogeneous differential equation.

Putting y = vx $\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$, given equation reduces to

$v+x\frac{dv}{dx}=-\frac{1-v\sin v}{\sin v}$

$\Rightarrow x\frac{dv}{dx}=-\frac{1-v\sin v}{\sin v}-v$

$\Rightarrow x\frac{dv}{dx}=-\frac{1}{\sin v}$

$\Rightarrow$ sin v dv = $-\frac{1}{x}$ dx, if x $\neq$ 0

Integrating both sides,

$\Rightarrow\int$ sin v dv = $-\int\frac{1}{x}$ dx

$\Rightarrow$ - cos v = - log |x| + C

$\Rightarrow$ - cos $\left(\frac{y}{x}\right)$ + log |x| = C ...(i)

It is given that y (1) = $\frac{\pi}{2}$ i.e., when x = 1, y = $\frac{\pi}{2}$.

Putting x = 1 and y = $\frac{\pi}{2}$ in (i), we get

$\Rightarrow$ -cos $\frac{\pi}{2}$ + log 1 = C $\Rightarrow$ C = 0

Putting C = 0 in (i), we get

-cos $\left(\frac{y}{x}\right)$ + log |x| = 0

$\Rightarrow$ log |x| = cos $\left(\frac{y}{x}\right)$

Hence, log |x| = cos $\left(\frac{y}{x}\right)$, is the required solution.

OR

The given functional relation is given by,

y = e^x cos x ...(i)

Differentiating both sides w.r.t x we have $\frac{dy}{dx}=e^x\frac{d}{dx}$ (cos bx) + cos bx $\frac{d}{dx}$ (e^x)

$\Rightarrow\frac{dy}{dx}$ = -e^x sin bx + e^xcos x = e^x [-sin bx + cos x] ...(ii)

Differentiating both sides w.r.t x we get $\frac{d^2y}{dx^2}=-\frac{d}{dx}$ (e^x sin x) + $\frac{d}{dx}$ (e^x cos x)

$\Rightarrow\frac{d^2y}{dx^2}$ = - {-e^x $\frac{d}{dx}$ (sin) + sin x $\frac{d}{dx}$ e^x} + e^x $\frac{d}{dx}$ (cos x) + cos x $\frac{d}{dx}$ e^x

$\Rightarrow\frac{d^2y}{dx^2}$ = -e^x cos x - e^x sin x - e^x sin x + e^x cos x

= e^x {-cos x - 2 sin x + cos x} ...(iii)

L.H.S. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}$ + 2y = 0

= e^x [-cos x - 2 sin x + cos x] -e^x [-sin x + cos x] + 2$e^x\cdot$ cos bx

= e^x [-cos x - 2 sin bx + cos x + 2 sin x - 2 cos x + 2 cos x] = $e^x\cdot$ 0 = 0 = R.H.S.

Hence given equation is a solution of given differential equation.

30. First, we will convert the given inequations into equations, we obtain the following equations:

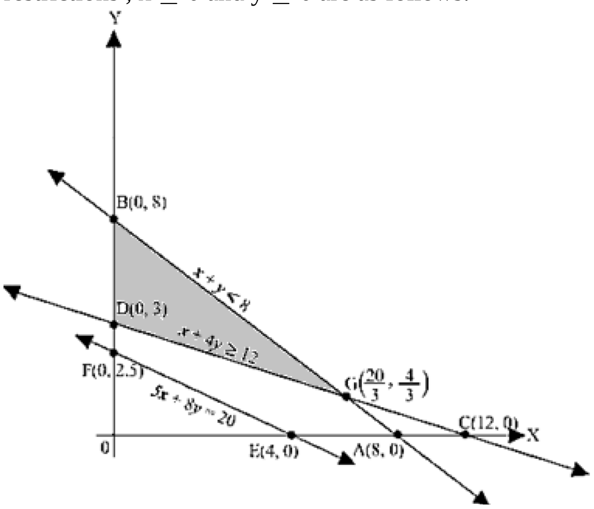
x + y = 8, x + 4 y = 12, x = 0 and y = 0

5 x + 8 y = 20 is already an equation.

Region represented by x + y $\leq$ 8 The line x + y = 8 meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line x + y = 8 . Clearly (0,0) satisfies the inequation x + y $\leq$ 8.50, the region in x y plane which contain the origin represents the solution set of the inequation x + y $\leq$ 8.

Region represented by x + 4 y $\geq$ 12 :

The line x + 4 y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4 y = 12 . Clearly (0,0) satisfies the inequation x + 4 y $\geq$ 12 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + 4 y $\geq$ 12.
The line 5 x + 8 y = 20 is the line that passes through E(4,0) and $F\left(0, \frac{5}{2}\right)$ Region represented by x $\geq$ 0 and y $\geq$ 0 : since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.
The feasible region determined by subject to the constraints arex + y $\leq$ 8, x + 4 y $\geq$ 12,5 x + 8 y = 20 and the non-negative restrictions , x $\geq$ 0 and y $\geq$ 0 are as follows.



The corner points of the feasible region are B(0,8), D(0,3), $G\left(\frac{20}{3}, \frac{4}{3}\right)$
The values of objective function at corner points are as follows:
Corner point: Z = 30x +20y
B(0,8): 160
D(0,3): 60
$G\left(\frac{20}{3}, \frac{4}{3}\right)$: 266.66
Therefore, the minimum value of objective function Z is 60 at the point D(0,3) . Hence, x = 0 and y = 3 is the optimal solution of the given LPP.
Thus, the optimal value of objective function Z is 60.

OR

Our problem is to mimmise and maximise the given objective function given as Z = x + 2 y .....(i)
Subject to the given constraints,
x + 2y $\geq$100 ....(ii)
2x - y $\leq$ 0 ....(iii)
2x + y $\leq$200 ....(iv)
x$\geq$ 0, y$\geq$ 0 ....(v)
Table for line x + 2y = 100 is

x	0	100
y	50	0

So, the line x + 2y = 100 is passing through the points with coordinates (0, 50) and (100, 0).
On replacing the coordinates of the origin O (0, 0) in the inequality x + 2y $\geq$100, we get
$2 \times 0 + 0 \geq 100$
$\Rightarrow \quad 0 \geq 100$ (which is False)
So, the half plane for the inequality of the line (ii) is away from the origin, which means that the point O(0,0) does not lie in the feasible region of the inequality of (ii)
Table for the line (iii) 2x - y = 0 is given as follows.

x	0	10
y	0	20

So, the line 2x - y = 0 is passing through the points (0, 0) and (10, 20).

On replacing the point (5, 0) in the inequality 2x - y$\leq$ 0, we get

$2 \times 5 - 0 \leq 0$

$\Rightarrow \quad 10 \leq 0$ (which is False)

So, the half plane for the inequality of (iii) is towards Y-axis.

Table of values for line 2x + y = 200 is given as follows.

x	0	100
y	200	0

So, the line 2x + y = 200 is passing through the points with coordinates (0, 200) and (100, 0).

On replacing O (0, 0) in the inequality 2 + y $\leq$ 200, we get

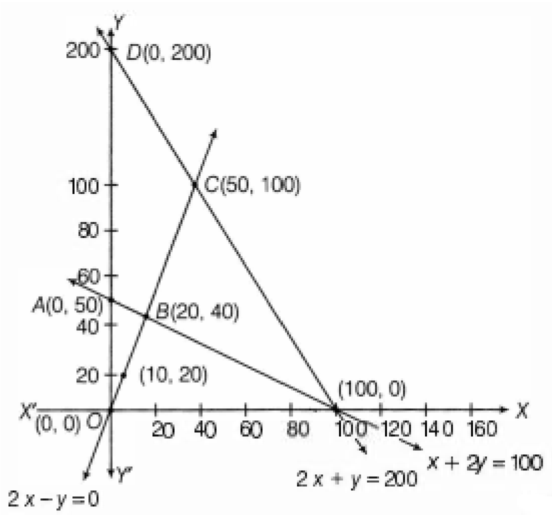
$2 \times 0 + 0 \leq 200$

$\Rightarrow \quad 0 \leq 200$ (which is true)

So, the half plane for the inequality of the line (iv) is towards the origin, which means that the point O (0,0) is a point in the feasible region.

Aslo, x, y$\geq$ 0

So, the region lies in the I quadrant only.



On solving equations 2x - y = 0 and x + 2y = 100, we get the point of intersection as B(20, 40).

Again, solving the equations 2x - y = 0 and 2x + y = 200, we get C(50, 100).

$\therefore$ Feasible region is ABCDA, which is a bounded feasible region.

The coordinates of the corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100) and D(0, 200).

The values of Z at corner points are given below:

Corner points	Z = x + 2y
A(0, 50)	Z = 0 + 2$\times$50 = 100
B(20, 40)	Z = 20 + 2$\times$40 = 100
C(50, 100)	Z = 50 + 2$\times$100 = 250
D(0, 200)	Z = 0 + 2$\times$200 = 400

The maximum value of Z is 400 at D(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining A(0, 50) and B(20, 40).

31. According to the question, $f(x) = |x - 3|$

To Check the continuity of f(x) at x = 3.

Here, LHL = $\lim\limits_{x \to 3^-} |x - 3| = \lim\limits_{h \to 0} |3 - h - 3|$

$= \lim\limits_{h \to 0} |-h| = 0$

RHL = $\lim\limits_{x \to 3^+} |x - 3| = \lim\limits_{h \to 0} |3 + h - 3|$

$= \lim\limits_{h \to 0} |h| = 0$

$and \ \ f(3) = |3 - 3| = 0$

$\therefore$, LHL = RHL = f(3)

Hence, f is continuous at x = 3.

To check the differentiability of f(x) at x = 3.

LHD = $f'\left(3^{-}\right)=\lim\limits_{h \rightarrow 0} \frac{f(3-h)-f(3)}{-h}$

$=\lim\limits_{h \rightarrow 0} \frac{|3-h-3|-|3-3|}{-h}$

$=\lim\limits_{h \rightarrow 0} \frac{|-h|}{-h}=\lim\limits_{h \rightarrow 0} \frac{h}{-h}=-1$

RHD = f'(3^+) = $\lim\limits_{h \rightarrow 0} \frac{f(3+h)-f(3)}{h}$

$=\lim\limits_{h \rightarrow 0} \frac{|3+h-3|-|3-3|}{h}$

$=\lim\limits_{h \rightarrow 0} \frac{|h|}{h}=\lim\limits_{h \rightarrow 0} \frac{h}{h}=1$

Since, LHD $\neq$RHD at x = 3.

$\therefore$ f(x) is not differentiable at x=3

Hence proved.

Section D

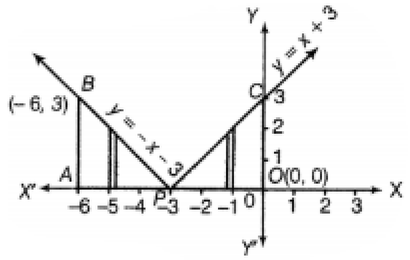
32. First, we sketch the graph of $y=|x+3|$

$\therefore y=|x+3|=\begin{cases}x+3, & \text{if } \quad x+3 \geq 0 \\ -(x+3), & \text{if } \quad x+3<0\end{cases}$

$\Rightarrow y=|x+3|=\begin{cases}x+3, & \text{if } \quad x \geq-3 \\ -x-3, & \text{if } \quad x<-3\end{cases}$

So, we have $y=x+3$ for x $\geq$ - 3 and y = -x - 3 for x < -3

A sketch of $y=|x+3|$ is shown below:



$y=x+3$ is the straight line which cuts X and Y-axes at (-3, 0 and (0, 3), respectively.

$\therefore y=x+3$ for x $\geq$ -3 represents the part of the line which lies on the right side of x = -3.

Similarly,$y=-x-3$, x < -3 represents the part of line $y=-x-3$, which lies on left side of x = -3

Clearly, required area = Area of region ABPA + Area of region PCOP

$=\int_{-6}^{-3}(-x-3) d x+\int_{-3}^0(x+3) d x$

$=\left[-\frac{x^2}{2}-3 x\right]_{-6}^{-3}+\left[\frac{x^2}{2}+3 x\right]_{-3}^0$

$=\left[\left(-\frac{9}{2}+9\right)-(-18+18)\right]+\left[0-\left(\frac{9}{2}-9\right)\right]$

$=\left(-\frac{9}{2}-\frac{9}{2}\right)+(9+9)$

= 18 - 9

= 9 sq. units

33. Given that A = {2, 3, 4}, B = {2, 5, 6, 7}
    1. Let $f: A \rightarrow B$ defined by
       f = {(x, y) : y = x + 3}
       i.e. f = {(2, 5), (3, -6), (4, 7)} f = {(2, 5), (3, 6), (4, 7)} which is an injective mapping.
    2. Let $g: A \rightarrow B$ denote a mapping such that g = {(2, 2), (3, 5), (4, 5) which is not an injective mapping.
    3. Let $h: B \rightarrow A$ denote a mapping such that h = {(2, 2), (5, 3), (6, 4), (7, 4)} which is a mapping from B to A.

OR

Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as (2, 2) $\notin$ R as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as (1, 39) $\in$ R but (39, 1) $\notin$ R

R is not transitive

as (11, 19) $\in$ R, (19, 3) $\in$ R

But (11, 3) $\notin$ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

34. In matrix form ,the system of equations

x-y+2z=7

3x + 4y - 5z = - 5

and 2x - y+ 3z = 12

can be written as,

AX=B ...(i)

where,

$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

Here, |A| =1 (12- 5) + 1 (9 + 10) +2(-3- 8)

=1 (7) + 1 (19) +2(-11) (1)

= 7 + 19 - 22 = 4

$\Rightarrow$ |A| $\neq$ 0

So, A is non-singular and its inverse exists.

Now, co-factors of elements of |A| are

$A_{11} = (-1)^2 \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 1(12 - 5) = 7$

$A_{12} = (-1)^3 \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -1(9 + 10) = -19$

$A_{13} = (-1)^4 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3 - 8) = -11$

$A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -1(-3 + 2) = 1$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3 - 4) = -1$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1(-1 + 2) = -1$

$A_{31} = (-1)^4 \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 1(5 - 8) = -3$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -1(-5 - 6) = 11$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 1(4 + 3) = 7$

$\therefore \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$= \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T$

$= \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

and $A^{-1} = \frac{\operatorname{adj}(A)}{|A|}$

$= \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

Therefore, from Eq. (i), we get,

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, comparing corresponding elements, we get x = 2, y = 1 and z = 3.

35. We are given that, equation of the line is $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} = \lambda$

General point on the line is given by $(3\lambda - 4, 5\lambda - 6, -2\lambda + 1)$ …… (1)

Another equation of line is

3x – 2y + z + 5 = 0

2x + 3y + 4z – 4 = 0

Suppose a, b, c be the direction ratio of the line so, it will be perpendicular to normal of 3x – 2y + z + 5 = 0 and 2x + 3y + 4z – 4 = 0

So, using a1a2 + b1b2 + c1c2 = 0

(3)(a) + (– 2)(b) + (1)(c) = 0

3a – 2b + c = 0 …… (2)

Again, (2)(a) + (3)(b) + (4)(c) = 0

2a + 3b + 4c = 0 …… (3)

Solving (2) and (3) by cross – multiplication,

$\frac{a}{(-2)(4)-(3)(1)} = \frac{b}{(2)(1)-(3)(4)} = \frac{c}{(3)(3)-(-2)(2)}$

$\frac{a}{-8-3} = \frac{b}{2-12} = \frac{c}{9+4}$

$\frac{a}{-11} = \frac{b}{-10} = \frac{c}{13}$

Direction ratios are proportional to – 11, – 10, 13

Let z = 0 so

3x – 2y = – 5 …… (i)

2x + 3y = 4 …… (ii)

Solving (i) and (ii) by eliminations method,

6 x - 4 y = -10

$\pm 6x \pm 9y = \pm 12$

– 13y = – 22

$y = \frac{22}{13}$

Substitute y in equation (i)

3x – 2y = – 5

3x – $2\frac{22}{13}$ = – 5

3x – $\frac{44}{13}$ = – 5

3x = – 5 + $\frac{44}{13}$

3x = $\frac{-21}{13}$

x = $\frac{-7}{13}$

since, the equation of the line (2) in symmetrical form,

$\frac{x+\frac{7}{13}}{-11} = \frac{y-\frac{22}{13}}{-10} = \frac{z-0}{13}$

Substitute the general point of a line from equation (1)

$\frac{3\lambda-4+\frac{7}{13}}{-11} = \frac{5\lambda-6-\frac{22}{13}}{-10} = \frac{-2\lambda+1}{13}$

$\frac{39\lambda-52+7}{-11\times13} = \frac{65\lambda-78-22}{-10\times13} = \frac{-2\lambda+1}{13}$

$\frac{39\lambda-45}{-11} = \frac{65\lambda-100}{-10} = \frac{-2\lambda+1}{1}$

The equation of the plane is 45x – 17y + 25z + 53 = 0

Hence the point of intersection is (2, 4, –3)

OR

Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

$\therefore$ Direction of line $\vec{b}$ = $(\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

= $-3\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore$ Equation of required line is

$\vec{r}$= $(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ .... (i)

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$(1 - 3\lambda)2$ + $(2 + 5\lambda)1$ + $(3 + 4\lambda)1$ = 4

$\Rightarrow \lambda$= 1

$\therefore$ Point of intersection is (4, -3, -1)

Section E

36. i. Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

    Let A denotes the event that daughter is at one end n(A) = 12 and B denotes the event that father, and mother are in the middle n(B) = 4

    Also, n(A ∩ B) = 4

    $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{4}{24}} = 1$

    ii. Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

    Let A denotes the event that mother is at right end. n(A) = 6 and B denotes the event that son and daughter are together. n(B) = 12

    Also, n (A ∩ B) = 4

    $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{12}{24}} = \frac{1}{3}$

    iii. Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

    Let A denotes the event that father, and mother are in the middle. n(A) = 4 and B denote the event that son is at right end. n(B) = 6

    Also, n (A ∩ B) = 2

    $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{24}}{\frac{6}{24}} = \frac{1}{3}$

    OR

    Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM},

    where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

    Let A denotes the event that father and son are standing together. n(A) = 12 and B denote the event that mother and daughter are standing together. n(B) = 12

    Also, n (A ∩ B) = 8$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{8}{24}}{\frac{12}{24}} = \frac{2}{3}$

37. i. Displacement between Ram's house and school = $\sqrt{4^2 + 3^2}$

    = $\sqrt{25}$

    = 5 km

    ii. Distance travelled to reach school by Ram = 4 + 3

    = 7 km

    iii. Position vector of school = $4\hat{i}$ + $3\hat{j}$

    Position vector of Suresh's

    = (4 + 6 cos30)$\hat{i}$ + (3 + 6 sin30)$\hat{j}$

$= (4 + \frac{6\sqrt{3}}{2})\hat{i} + (3 + \frac{6\times1}{2})\hat{j}$

= (4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$

Vector distance from school to Suresh's home

[(4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$] - [(4$\hat{i}$ + 3$\hat{j}$)]

$3\sqrt{3}\hat{i} + 3\hat{j}$

OR

Position vector of Ram's house = 0$\hat{i}$ + 0$\hat{j}$

Position vector of Suresh's house = (4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$

$\therefore$ Displacement from Ram's house to suresh's house

= (4 + $3\sqrt{3}$)$\hat{i}$ + 6$\hat{j}$ - (0$\hat{i}$ + 0$\hat{j}$)

$(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

38. i. c(h) = 100 h + 320 + $\frac{1600}{h}$

Let l ft be the length and h ft be the height of the tank. Since breadth is equal to 5 ft. (Given)

$\therefore$ Two sides will be 5h sq. feet and two sides will be lh sq. feet. So, the total area of the sides is (10 h + 2lh)ft^2

Cost of the sides is ₹10 per sq. foot. So, the cost to build the sides is (10h + 2Ih) $\times$ 10 = ₹(100h + 20Ih)

Also, cost of base = (5 l) $\times$ 20 = ₹100 l

$\therefore$ Total cost of the tank in ₹ is given by c = 100 h + 20 lh + 100l

Since, volume of tank = 80 ft^3

$\therefore$ 5lh = 80 ft^3 $\therefore l = \frac{80}{5h} = \frac{16}{h}$

$\therefore$ c(h) = 100h + $20\left(\frac{16}{h}\right)h + 100\left(\frac{16}{h}\right)$

= 100h + 320 + $\frac{1600}{h}$

ii. C(h) = 100h + 320 + $\frac{1600}{h}$

$\frac{dC(h)}{dh} = 100 - \frac{1600}{h^2}$

$\frac{d^2C(h)}{dh^2} = -\left(\frac{-2}{h^3}\right)1600$

at h = 4

$\frac{d^2C(h)}{dh^2}$ = 50 > 0

Hence cost is minimum when h = 4 ft

iii. To minimize cost, $\frac{dc}{dh}$ = 0

$\Rightarrow$ 100 - $\frac{1600}{h^2}$ = 0

$\Rightarrow$ $100h^2$ = 1600 $\Rightarrow$ h^2 = 16 $\Rightarrow$ h = $\pm$4

$\Rightarrow$ h = 4 [$\because$ height can not be negative]

OR

Minimum cost of tank is given by

c(4) = 400 + 320 + $\frac{1600}{4}$

= 720 + 400 = ₹1120