Solution

MATHEMATICS

Class 10 - Mathematics

Section A

1.

(c) 45 Explanation: We have, $135 = 3 \times 45$ $= 3 \times 3 \times 15$ $= 3 \times 3 \times 3 \times 5$ $= 3^3 \times 5$ Now, for 225 will be $225 = 3 \times 75$ $= 3 \times 3 \times 5 \times 5$ $= 3^2 \times 5^2$

The HCF will be $3^2 \times 5 = 45$

2.

(c) 2

Explanation:

No. of zeros = no. of times the graph cuts the x-axis. Here the graph cuts the x-axis two times so no. of zero's = 2

3. (a) no solution

Explanation:

Since, we have y = 0 and y = -6 are two parallel lines. therefore, no solution exists.

4.

(c) pq Explanation:

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Let the roots of given quadratic equation be \alpha and \beta.
On comparing equation x^2 - (p - q)x + k = 0
with ax^2 + bx + c = 0, we have
a = 1, b = -(p + q), c = k
We know that
\Rightarrow \alpha + \beta = \frac{-b}{a}
Put the value a and b
\Rightarrow \alpha + \beta = \frac{p+q}{1}
\Rightarrow \alpha + \beta = p + q \dots (i)
Given \alpha = p
Put the value of \alpha in equation (i),
\Rightarrow p + \beta = p + q
\Rightarrow \beta = q
But we know that
\alpha \cdot \beta = \frac{c}{a}
Put the values
p.q. =\frac{k}{1}
Then, k = pq.
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5.

(d) 10Explanation:AP can be written as49, 46, 43...

a = 49, d = -3 n = 14 $a_{14} = a + 13d$ = 49 + 13(-3) = 49 - 39 $a_{14} = 10$

 14^{th} term from last = 10

6.



7.

(d) $\sqrt{41}$ units Explanation: OP = $\sqrt{(4-0)^2 + (0-(-5))^2}$ = $\sqrt{(16+25)}$ = $\sqrt{41}$ units

8.

(d) 8 cm

Explanation:

As PQ \parallel AC by using proportionality theorem

$$\Rightarrow \frac{BP}{PA} = \frac{BQ}{QC}$$
$$\Rightarrow \frac{4}{2.4} = \frac{5}{QC}$$
$$\Rightarrow QC = \frac{5 \times 2.4}{4} = 5 \times 0.6$$

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\Rightarrow QC = 3 cm
\therefore BC = BQ + QC
= 5 + 3
= 8 cm
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9.

(c) 25^o **Explanation:**

PA = PB (tangents from same point) PAB is an isosceles triangle angle PAB = angle PBA = xIn triangle PBA $50^{\circ} + x + x = 180^{\circ}$ 2x = 130° x = 65° Now $\angle PAO = 90^{\circ}$ x + ∠OAB = 90° $\angle OAB = 90^{\circ} - 65^{\circ}$ ∠OAB = 25°

10.

(b) $\sqrt{125}$

 $\mathrm{OQ} \perp \mathrm{QP}$

Explanation:

Length of a tangent to the circle from an external point = 10 cm Dadius(n) = F- OD - D

In right $\triangle OPQ$, $OP^2 = OQ^2 + QP^2$ (Pythagoras Theorem) = $(5)^2 + (10)^2 = 25 + 100 = 125$ $OP = \sqrt{125} \text{ cm}$

11.

(c) tan²A **Explanation:** $1{+}{\rm tan^2}\,A$ $1 + \cot^2 A$ 1+ $1 + \tan^2 A$ $\tan^2 A + 1$ $\tan^2 A$ $=\left(1+ an^2A
ight)\left(rac{ an^2A}{ an^2A+1}
ight)= an^2A$

Hence, the correct choice is tan²A.

(a) 9 12. **Explanation:**

$$2 \sin^2 30^0 + 3 \tan^2 60^0 - \cos^2 45^0$$
$$= 2 \times \left(\frac{1}{4}\right) + 3 \times 3 - \frac{1}{2} = 9$$

13.

(c) 7 m **Explanation:**

A
B
B
T
C
height of tree = AB
tan
$$45^{\circ} = \frac{AB}{BC}$$

 $1 = \frac{AB}{7}$
AB = 7 m

14.

(d) 126° **Explanation:**

We have given that area of the sector is $\frac{7}{20}$ of the area of the circle.

Therefore, area of the sector $=\frac{7}{20} \times$ area of the circle $\therefore \frac{\theta}{360} \times \pi r^2 = \frac{7}{20} \times \pi r^2$ Now we will simplify the equation as below, $\frac{\theta}{360} = \frac{7}{20}$ Now we will multiply both sides of the equation by 360, $\therefore \theta = \frac{7}{20} \times 360$ $\therefore \theta = 126$ Therefore, sector angle is 126° .

15.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm $\Rightarrow \frac{\theta}{360^{\circ}} \times 2\pi r = 20$ $\Rightarrow \frac{60^{\circ}}{360^{\circ}} \times 2\pi r = 20$ $\Rightarrow \frac{\pi r}{3} = 20$ $\Rightarrow r\left(\frac{\pi}{3}\right) = 20$ $\Rightarrow r\left(\frac{\pi}{3}\right) = 20$ \Rightarrow r = $\frac{60}{\pi}$ cm

16.

(c) $\frac{1}{6}$ **Explanation:** When two dice are thrown, The sample space S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) ∴ n(S) = 36 Let A: Event of getting the numbers whose difference is 3. ∴ A = (1, 4), (2, 5), (3, 6), (4, 1), (5, 2), (6, 3)} n(A) = 6 ∴ Required probability = $\frac{n(A)}{n(S)}$ = $\frac{6}{36}$ = $\frac{1}{6}$

17.

(c) $\frac{1}{52}$

Explanation:

A card is drawn at random from a pack of well shuffled 52 playing cards. 'S' is the sample space.

 \therefore n(S) = 52

Event A: The card drawn is an ace of spade

 $\therefore n(A) = 1$ $\therefore P(A) = \frac{n(A)}{n(S)}$ $= \frac{1}{52}$

18.

19.

(d) 24.5 Explanation: Median = 26 Mode = 29 Mode = 3Median - 2Mean Hence, $Mean = \frac{3Median - Mode}{2}$ $= \frac{3(26) - 29}{2}$ $= \frac{78 - 29}{2}$ $= \frac{49}{2}$ = 24.5(d) A is false but R is true.

Explanation:

A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

21. We have to find the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively. Let assume that x be the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively. So, it meansx divides 85 - 1 = 84

x divides 85 - 1 = 84 and x divides 72 - 2 = 70 So, from this we concluded that = x divides 84 and 70 = x = HCF (84, 70) Now, to find HCF(84, 70), we use method of prime factorization. Prime factors of $84 = 2 \times 2 \times 3 \times 7$ Prime factors of $70 = 2 \times 5 \times 7$ So, = HCF (84, 70) = $2 \times 7 = 14$ = x = 14

Hence, 14 is the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

22. Since G is the mid-point of PQ, then,

PG = GQ. According to the question, GH || QR.Therefore by basic theorm of proportionality, we have, $\frac{PG}{CO} = \frac{PH}{HP}$

 $\frac{PG}{GQ} = \frac{PH}{HR}$ $1 = \frac{PH}{HR} \text{ [as PG = GQ]}$ PH = HR

Hence, H is the mid-point of PR.

23. Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.



Given that, Radius of circle = 7 cm Central angle = 90° Now, area of minor sector of circle $=\frac{\pi r^2\theta}{360^\circ}$ $-\frac{\pi(7)^2}{2}$ $=\frac{4}{22\times7\times7}$ $= 38.5 \text{ cm}^2$ Area of complete circle $=\pi r^2$ $=\pi(7)^{2}$ $= 154 \text{ cm}^2$ Now, area of major sector = Area of complete circle - Area of minor sector = 154 - 38.5 $= 115.5 \text{ cm}^2$ Section C 26. Given numbers are 156, 208 and 260. Here, 260 > 208> 156 Let us find the HCF of 260 and 208, By using Euclid's division lemma for 260 and 208, we get $260 = (208 \times 1) + 52$ Here, the remainder is 52, not zero. On taking 208 as new dividend and 52 as new divisor and then apply Euclid's division lemma, we get $208 = (52 \times 4) + 0$ Here, the remainder is zero and the divisor is 52. So, HCF of 208 and 260 is 52. Now, 156 >52 Let us find the HCf of 52 and 156. By using Euclid's division lemma , we get $156 = (52 \times 3) + 0$ Here, the remiander is zero and the divisor is 52. So, HCF of 52 amd 156 is 52. Thus, HCf of 156, 208 and 260 is 52. Hence, the minimum number of buses $= \frac{156}{52} + \frac{208}{52} + \frac{260}{52} = \frac{156 + 208 + 260}{52} = \frac{624}{52} = 12$ The minimum number of buses is 12. 27. Let the given polynomial is $p(x) = 4x^2 + 4x + 1$ Since, α , β are zeroes of p(x), $\therefore \alpha + \beta = \text{sum of zeroes} = \frac{-4}{4}$ Also, α . β = Product of zeroes = α . $\beta = \frac{1}{4}$ Now a quadratic polynomial whose zeroes are 2α and 2β x^2 - (sum of zeroes)x + Product of zeroes $=x^2-(2lpha+2eta)x+2lpha imes2eta$ $=x^2-2(lpha+eta)x+4(lphaeta)$ $=x^2-2 imes(-1)x+4 imesrac{1}{4}$ $= x^{2} + 2x + 1$ The quadratic polynomial whose zeroes are 2α and 2β is $x^2 + 2x + 1$

28. Since each prize is Rs 20 less than its preceding prize,

therefore, the value of the seven successive cash prizes will form an AP.

Let the first prize be Rs a. Then the winner prizes, in succession, will be Rs (a - 20), Rs (a - 40), Rs (a - 60), etc. Here, A = ad = (a - 20) - a = -20n = 7 $S_n = 700$ We know that $S_n = rac{n}{2} [2A(n-1)d]$ $\Rightarrow 700 = \frac{7}{2} [2a + (7 - 1)(-20)]$ $\Rightarrow 700 = \frac{7}{2}[2a - 120]$ $\Rightarrow 700 = 7(a - 60)$ $\Rightarrow a - 60 = \frac{700}{7}$ \Rightarrow a - 60 = 100 \Rightarrow a = 100 + 60 \Rightarrow a = 160 \Rightarrow Value of first prize = Rs 160 Value of second prize = Rs 160 - Rs 20 = Rs 140 Value of third prize = Rs 140 - Rs 20 = Rs 120 Value of fourth prize = Rs 120 - Rs 20 = Rs 100 Value of fifth prize = Rs 100 - Rs 20= Rs 80 Value of sixth prize = Rs 80 - Rs 20 = Rs 60 Value of seventh prize = Rs 60 - Rs 20= Rs 40 OR The given AP is: 9,12, 15, 18..... Here a = 9 and d = 12 - 9 = 3Let nth term of this A.P. is 39 more than 26th term $a_n = a_{36} + 39$ a + (n - 1)d = a + (36 - 1)d + 39 $9 + (n - 1) \times 3 = 9 + 35 \times 3 + 39$ 9 + 3n - 3 = 9 + 105 + 393n=9+144-9+3 3n=147 $n = \frac{147}{3} = 49$ Therefore, 49th term of A.P. will be 39 more than its 36th term.

29. According to question we are given that PQ = 10 cm, QR = 8 cm and PR = 12 cm.

We know that the lengths of the tangents drawn from an external point to a circle are equal.

Let PL = PN = x; QL = QM = y; RM = RN =z. Now,PL + QL = PQ \Rightarrow x+y = 10, ...(i) QM + RM = QR \Rightarrow y + z = 8, ...(ii) Subtracting (ii) from (iii), we get x - y = 4. ...(iv) Solving (i) and (iv), we get x = 7, y = 3. Substituting y = 3 in (ii), we get z = 5 \therefore QM = y = 3 cm, RN = z = 5 cm, PL = x = 7 cm.

Given A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: $\angle PTQ = 2 \angle OPQ$ Proof: Let \angle PTQ = θ Since TP, TQ are tangents drawn from point T to the circle. TP = TQ \therefore TPQ is an isoscles triangle $\therefore \angle \text{TPQ} = \angle \text{TQP} = \frac{1}{2} (180^\circ - \theta) = 90^\circ - \frac{\theta}{2}$ Since, TP is a tangent to the circle at point of contact P $\therefore \angle OPT = 90^{\circ}$ $\therefore \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - (90^\circ - \frac{1}{2}\theta) = \frac{\theta}{2} = \frac{1}{2} \angle PTQ$ Thus, $\angle PTQ = 2 \angle OPQ$ 30. Given, $\tan\theta + \sin\theta = m....(1)$ &, $\tan\theta - \sin\theta = n$ (2) Now, LHS = $m^2 - n^2$ = $(\tan \theta + \sin \theta)^2$ - $(\tan \theta - \sin \theta)^2$ [from (1) & (2)] $= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$ = 4 tan $\theta \sin \theta$ $=4\sqrt{\tan^2\theta\cdot\sin^2\theta}$ $=4\sqrt{ an^2 heta(1-\cos^2 heta)}$ $=4\sqrt{\tan^2 heta-\sin^2 heta}$ $=4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$ $=4\sqrt{mn}$ [from (1) & (2)] = RHS. Hence, Proved. 31

L.	Class	Mid-point (x _i)	f _i	$\mathbf{d_i} = \mathbf{x_i} - \mathbf{A}$	f _i d _i
	10-15	12.5	4	-10	-40
	15-20	17.5	10	-5	-50
	20-25	22.5=A	5	0	0
	25-30	27.5	6	5	30
	30-35	32.5	5	10	50
	Total		$\sum f_i = 30$		$\sum f_i d_i = 10$

Here, assumed mean, A = 22.5

Mean =
$$A + \frac{\sum f_i d_i}{\sum f_i}$$

= 22.5 + $\frac{-10}{30}$
= 22.5 - 0.33

32. Let the present age of Roohi be x years. Then, Section D

3 years ago, Roohi's age = (x - 3) years 5 years from now, Roohi's age = (x + 5) years It is given that $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$ \Rightarrow 6x + 6 = x² + 2x - 15 $\Rightarrow x^2 - 4x - 21 = 0$ $\Rightarrow x^2 - 7x + 3x - 21 = 0$ \Rightarrow x(x - 7) + 3(x - 7) = 0 \Rightarrow x - 7 = 0 or x + 3 = 0 Since age cannot be negative, $x \neq -3$ \Rightarrow x = 7 Hence, Roohi's present age is 7 years. OR Let $\triangle ABC$ be the right angle triangle, right angled at B, as shown in figure. B < a Also, let AB = c cm, BC = a cm and AC = b cm Then, according to the given information, we have $b = 6 + 2a \dots (i)$ (Let a be the shortest side) and c = 3a - 6We know that, $b^2 = c^2 + a^2$ $\Rightarrow (6 + 2a)^2 = (3a - 6)^2 + a^2 \dots [Using (i) and (ii)]$ $\Rightarrow 36 + 4a^2 + 24a = 9a^2 + 36 - 36a + a^2$ $\Rightarrow 60a = 6a^2$ \Rightarrow 6a=60 ...[:: a cannot be zero] \Rightarrow a = 10 cm Now, from equation (i), $b = 6 + 2 \times 10 = 26$ and from equation (ii), $c = 3 \times 10 - 6 = 24$ Thus, the dimensions of the triangle are 10 cm, 24 cm and 26 cm.

33. Height of the statue, SP = 1.46 m(given)

Suppose PB, the height of the pedestal = h metre

According to question angles of elevation of S and P are $60^\circ \text{and} \ 45^\circ$ respectively.



Clearly $r = \frac{1}{2}$, $h = \frac{1}{2}$ Volume of solid = Volume of Cone + Volume of Hemisphere $= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$ $= \frac{1}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{2}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{3} \times 3.14 \times \frac{1}{2} \times \frac{1}{2} \times [\frac{1}{2} + 2(\frac{1}{2})]$ $= \frac{1}{3} \times \frac{3.14}{4} \times \frac{3}{2}$ $= \frac{1.57}{4} = \frac{157}{400} \text{ cm}^3 \text{ or } 0.3925 \text{ cm}^3$

35.	C.I.	x _i	f	$\mathbf{u_i} = \frac{\mathbf{x_i} - \mathbf{a}}{\mathbf{h}}$	f _i u _i
	10 - 25	17.5	2	-2	-4
	25 - 40	32.5	3	-1	-3
	40 - 55	47.5 a	7	0	0

55 - 70	62.5	6	1	6
70 - 85	77.5	6	2	12
85 - 100	92.5	6	3	18
		30		29

 $\overline{\text{Mean} = 47.5 + \frac{29}{30} \times 15}$

= 47.5 + 14.5 = 62

36. i. x + y + 2 = 15

Section E

x + y = 13 ...(i) Area of bedroom + Area of kitchen = 95 $5 \times x + 5 \times x + 5 \times y = 95$ 2x + y = 19 ...(ii) In $\triangle ABD$ $\tan 60^{\circ} = \frac{120\sqrt{3}}{27}$ BD $BD = \frac{120\sqrt{3}}{\sqrt{3}}$ BD = 120 m <u>30</u>σ ٦ 120 🕹 c^{<u>⁄30</u>°} Ď In $\triangle ABC$ $\tan 30^{\circ} = \frac{AB}{BC}$ $\underline{1} - \frac{120\sqrt{3}}{2}$ $\frac{1}{\sqrt{3}}$ BCBC = 360 m \therefore CD = BC - BD = 360 - 120 = 240 m ii. Length of outer boundary = 12 + 15 + 12 + 15 = 54 m x + y = 132x + y = 19iii. -x = -6 x = 6 Area of bedroom 1 = 5 \times x $= 5 \times 6 = 30 \text{ m}^2$ OR Area of living room = $(5 \times 2) + (9 \times 7)$ = 10 + 63 $= 73 \text{ m}^2$

37. 1.
$$43n$$

Distance covered in 2 sec = 2 m
length of shadow = 1 m
Total distance from base = 2 + 1 = 3m
 $\frac{1}{3} = \frac{\log 4t}{65} \frac{tr}{totam}$
 $\frac{1}{3} = \frac{\log 4t}{65} \frac{tr}{totam}$
 $\frac{1}{3} = \frac{\log 4t}{65} \frac{tr}{totam}$
 $\frac{1}{5} = \frac{\log 4t}{65} \frac{tr}{totam}$
 $\frac{1}{5} = \frac{\log 4t}{65} \frac{tr}{totam}$
 $\frac{1}{5} = \frac{1}{5} \frac{t}{5}$
 $\frac{1}{2} \frac{t}{4.5} = \frac{1}{5}$
 $\frac{1}{2} \frac{t}{4.5} = \frac{1}{5}$
Length of PQ = 200 - (.200) = 400
 $\frac{t}{16} \frac{t}{160} \frac{t}{200} \frac{t}{160} \frac{t}{200}$
 $\frac{t}{100} \frac{t}{2} \frac{t}{2} \frac{t}{2000} \frac{t}{2} = \frac{t}{200} \frac{t}{2000} \frac{t}{2000} \frac{t}{2} = \frac{t}{160,000} \frac{t}{100} \frac{t}{$

 $\Rightarrow 800\text{K} = 400\text{K} + 400$ $\Rightarrow 400\text{K} = 400$ $\Rightarrow \text{K} = 1$