Solution

MATHEMATICS

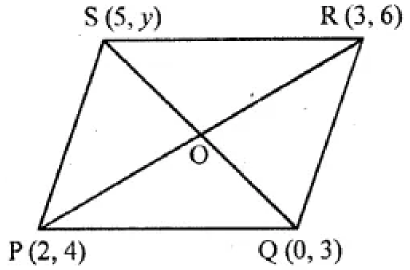
Class 10 - Mathematics

Section A

1. **(a)** 12
   Explanation:
   $2520 = 2^3 \times 3^2 \times 5 \times 7$
   on comparing
   a = 2, b = 5
   So,
   a + 2b = 2 + 2 $\times$ 5
   = 12
2. **(b)** 1
   Explanation:
   We see that the graph cuts the x-axis at 1 point which implies p(x) is zero at this 1 point only.
3. **(b)** consistent with unique solution.
   Explanation:
   Since the lines in the graph are not parallel, they will be consistent, also they are not coinciding, that means they have unique solution.
4. **(c)** -14
   Explanation:
   x = 2 is solution
   p(2) = $5(2)^2$ - 4(2) + (2 + k)
   0 = 20 - 8 + 2 + k
   k = -14
5. **(a)** -36
   Explanation:
   -36
6. **(c)** (2, 5)
   Explanation:
   Let cordinate of A(x,y)
   Then cordinate of mid point are $\left[\frac{(x-2)}{2}, \frac{(y+3)}{2}\right]$
   On comparing the cordinates of mid points
   $\frac{(x-2)}{2} = 0$
   x = 2
   $\frac{(y+3)}{2} = 4$
   y = 5
   Cordinates of A are (2, 5).
7. **(b)** 7

Explanation:

P(2, 4), Q(0, 3), R(3, 6) and S(5, y) are the vertices of parallelogram PQRS



Join PR and QS which intersect at O

$\therefore$ O is mid-point of PR and QS

When O is mid point of PR then coordinates

of O will be = $\left(\frac{2+3}{2}, \frac{4+6}{2}\right)$

$= \left(\frac{5}{2}, 5\right)$

O is mid-point of QS

$\therefore 5 = \frac{y+3}{2} \Rightarrow y + 3 = 10$

$\Rightarrow$ y = 10 - 3 = 7

8.

(c) $\frac{9}{2}$

Explanation:

According to Basic Proportionality theorem.

$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

$\frac{2}{3} = \frac{3}{x}$

x = $\frac{9}{2}$

9.

(c) 8.4 cm

Explanation:

AP = QP ...(1) (tangent from p)

also

AP = PR ...(2) (Tangent from p)

from (1) and (2)

AP =QP = PR = 4.2 cm

Now

QR = QP + PR

QR = 4.2 + 4.2

QR = 8.4 cm

10. **(a)** 24 cm

Explanation:

We know that, a tangent to a circle is perpendicular to the radius at the point of contact.

So, $\triangle$OCB is right a triangle, right angled at C.

Hence, by Pythagoras' theorem, we have:

$BC^2 = OB^2 - OC^2$

$\Rightarrow BC^2 = 225 - 81 = 144$

BC = 12 cm

We also know that, the tangents drawn from the same external point to a circle are equal.

Since BC and BD are tangents drawn from the same external point, B, we have:

BC = BD = 12 cm.

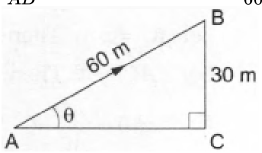
So, BC + BD = 24 cm.
Hence, BC + BD = 24 cm.

11.

(b) 60^{o}
Explanation:
$\tan^2\theta = 3$
$\tan\theta = \sqrt{3}$
$\tan\theta = \tan 60^\circ$
$\theta = 60^\circ$

12. **(a)** 9
Explanation:
9 ($\sec^2$ A - $\tan^2$ A)
= 9 $\times$ 1... ($\sec^2$ A - $\tan^2$ A=1)
= 9

13. **(a)** 30°
Explanation:
Let AB be the tower and B be the kite.
Let AC be the horizontal and let BC $\perp$ AC.
Let $\angle$CAB = θ.
BC = 30 m and AB = 60 m. Then,
$\frac{BC}{AB}$ = sin $\theta \Rightarrow$ sin θ = $\frac{30}{60} = \frac{1}{2} \Rightarrow$ sin θ = sin $30^{o} \Rightarrow \theta = 30^{o}$.

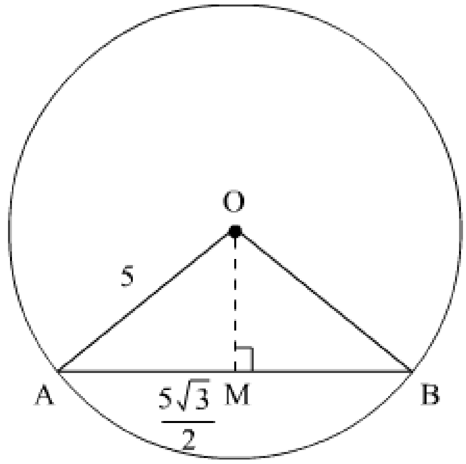


14.

(b) $\frac{132}{7}\text{cm}^2$
Explanation:
Angle of the sector is 60°
Area of sector = $(\frac{\theta}{360^o}) \times \pi r^2$
$\therefore$ Area of the sector with angle 60^{o} = $(\frac{60^o}{360^o}) \times \pi r^2$ cm^2
= $(\frac{36}{6})\pi$ cm^2
= 6 $\times$ $(\frac{22}{7})$ cm^2
= $\frac{132}{7}$ cm^2

15. **(a)** $\frac{25\pi}{3}\text{cm}^2$
Explanation:

We have to find the area of the sector OAB.



We have,

$AM = \frac{5\sqrt{3}}{2}$

So,

$\sin \angle AOM = \frac{5\sqrt{3}}{2(5)}$

Hence,

$\angle AOM = 60^\circ$

$\Rightarrow \angle AOB = 120^\circ$

Area of sector AOB $= \frac{120}{360} \times \pi \times 5^2 cm^2 = \frac{25\pi}{3}\ cm^2$

16.

(d) $\frac{4}{45}$

Explanation:

Number of disc in a box = 90

Numbered on it are 1 to 90 Prime numbers less than 23 are = 2, 3, 5, 7, 11, 13, 17, 19 = 8

Probability of a number being a prime less than $23 = \frac{8}{90} = \frac{4}{45}$

17.

(c) 0

Explanation:

An event which has no chance of occurrence is called an impossible event.

for example: The probability of getting more than 6 when a die is thrown is an impossible event because the highest number in a die is 6

The probability of an impossible event is always 0.

18.

(b) 12

Explanation:

Given,

mode - median = 24

median - mean = ?

we know that,

mode = 3 median - 2 mean

mode = median + 2 median - 2 mean

mode - median = 2 median - 2 mean

24 = 2 (median - mean)

median - mean = $\frac{24}{2}$ = 12

19.

(d) A is false but R is true.

Explanation:
A is false but R is true.

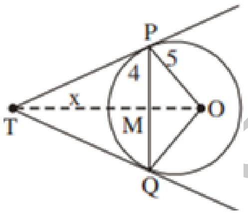
20.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
For 2k + 1, 3k + 3 and 5k - 1 to form an AP
(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)
k + 2 = 2k - 4
2 + 4 = 2k - k = k
k = 6
So, both assertion and reason are correct but reason does not explain assertion.

Section B

21. 26 = 2 $\times$ 13
91 = 7 $\times$ 13
HCF = 13
LCM = 2 $\times$ 7 $\times$ 13 = 182

22. In $\triangle ABC$, $AB \| DE$.
$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB}$ ...(i) [by Thales' theorem]
In $\triangle CDB$, $BD \| EF$
$\therefore \quad \frac{CF}{FD} = \frac{CE}{EB}$ ...(ii) [by Thales' theorem]
From (i) and (ii) we get
$\frac{CD}{DA} = \frac{CF}{FD}$
$\Rightarrow \quad \frac{DA}{DC} = \frac{FD}{CF}$ [taking reciprocals]
$\Rightarrow \quad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$
$\Rightarrow \frac{DA+DC}{DC} = \frac{FD+CF}{CF}$
$\Rightarrow \quad \frac{AC}{DC} = \frac{DC}{CF}$
$\Rightarrow \quad DC^2 = CF \times AC$

23. Join OT and OQ.
TP = TQ



$\therefore$ TM $\perp$ PQ and bisects PQ
Hence PM = 4 cm
Therefore OM = $\sqrt{25 - 16}$ = $\sqrt{9}$ = 3cm.
Let TM = x
From $\triangle$PMT, $PT^2 = x^2 + 16$
From $\triangle$POT, $PT^2 = (x + 3)^2 - 25$
Hence $x^2 + 16 = x^2 + 9 + 6x - 25$
$\Rightarrow$ 6x = 32 $\Rightarrow$ x = $\frac{16}{3}$
Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$
$\therefore$ PT = $\frac{20}{3}$ cm.

24. $\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$
L.H.S. $= \frac{1-\cos\theta}{1+\cos\theta}$
$= \frac{(1-\cos\theta)\times(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ [Multiplying and dividing by $(1 - \cos\theta)$]

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta} [\because 1 - \cos^2\theta = \sin^2\theta]$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \left[\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right]^2$

$= (cosec\theta - \cot\theta)^2 \quad \left[\because \frac{1}{\sin\theta} = cosec\theta, \frac{\cos\theta}{\sin\theta} = \cot\theta\right]$

= R.H.S. proved.

OR

$\text{LHS} = (\text{cosec A} - \cot \text{A})^2$

$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2 = \left(\frac{1-\cos A}{\sin A}\right)^2$

$= \frac{(1-\cos A)^2}{\sin^2 A} = \frac{(1-\cos A)^2}{1-\cos^2 A}$

$= \frac{1-\cos A}{1+\cos A} = \text{RHS}$

25. Long hand makes 24 rounds in 24 hours

Short hand makes 2 round in 24 hours

radius of the circle formed by long hand = 6 cm.

and radius of the circle formed by short hand = 4 cm.

Distance travelled by long hand in one round = circumference of the circle = $2 \times \pi \times r$

= $2 \times 6 \times \pi$

= 12π cm

Distance travelled by long hand in 24 rounds$= 24 \times 12\pi$

$= 288\pi$

Distance travelled by short hand in a round = $2 \times \pi \times r$

=$2 \times 4\pi$

= 8π cm

Distance travelled by short hand in 2 round

$= 2 \times 8\pi$

$= 16\pi$ cm

Sum of the distances = $288\pi + 16\pi = 304\pi$

$= 304 \times 3.14$

$= 954.56$ cm.

Thus, the sum of distances travelled by their tips in 24 hours is 954.56 cm.

OR

Given 3 horses are tethered with 7 m long ropes at three corners of $\triangle$ABC

Here radius of sectors, r = 7 m

Given sides of $\triangle$ABC are AB = 20 cm, BC = 30 m, CA = 40 m

Area of the plot which can be grazed = $\frac{x^\circ}{360^\circ} \times \pi r^2 + \frac{y^\circ}{360^\circ} \times \pi r^2 + \frac{z^\circ}{360^\circ} \times \pi r^2$

= $\frac{\pi r^2}{360}$[x + y + z]

= $\frac{\pi r^2}{360} \times 180$ [$\therefore$ x + y + z = 180]

= $\frac{1}{2}\pi r^2$

= $\frac{1}{2} \times \frac{22}{7} \times 7 \times 7$= 77 sq. m.

Section C

26. The greatest number of cartons is the HCF of 144 and 90

Now the prime factorization of 144 and 90 are

$144 = 16 \times 9 = 2^4 \times 3^2$.

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

HCF (144,90)= 2×3^2 = 18

$\therefore$ The greatest number of cartons each stack would have= 18.

27. Let p(x) = x^2 - 2x - (7p + 3)

Since -1 is a zero of p(x). Therefore,

p(-1) = 0

$(-1)^2$ - 2(-1) - (7p + 3)=0

1 + 2 - 7p - 3 = 0

3 - 7p - 3 = 0
7p = 0
p = 0
Thus, p(x) = x^2 - 2x - 3
For finding zeros of p(x), we put,
p(x) = 0
x^2 - 2x - 3 = 0
x^2 - 3x - x - 3 = 0
x(x-3) + 1(x-3) = 0
(x-3)(x+1) = 0
Put x - 3 = 0 and x + 1 = 0, we get,
Thus, x = 3, -1
Thus, the other zero is 3.

28. S_1 = 1 + 2 + 3 + ...n
S_2 = 1 + 3 + 5 + ...upto n terms
S_3 = 1 + 4 + 7 + ...upto n terms
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_1 = \frac{n}{2}[2(1) + (n-1)1]$
$S_1 = \frac{n}{2}[2 + n - 1]$
or, $S_1 = \frac{n(n+1)}{2}$
Also, $S_2 = \frac{n}{2}[2 \times 1 + (n-1)2]$
$S_2 = \frac{n}{2}[2 + 2n - 2]$
$= \frac{n}{2}[2n] = n^2$
and $S_3 = \frac{n}{2}[2 \times 1 + (n-1)3]$
$S_3 = \frac{n}{2}[2 + 3n - 3]$
$= \frac{n(3n-1)}{2}$
Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$
$= \frac{n[n+1+3n-1]}{2}$
$= \frac{n[4n]}{2}$
= $2n^2$ = $2S_2$

Hence Proved.

OR

The given AP is 9, 17, 25,...
Here, a = 9
d = 17 - 9 = 8
Let n terms of the AP must be taken
Then, S_n = 636
$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 636$
$\Rightarrow \frac{n}{2}[2(9) + (n-1)8] = 636$
⇒ n[9 + (n - 1)4] = 636
⇒ n[9 + 4n - 4] = 636
⇒ n[(4n + 5)] = 636
⇒ $4n^2$ + 5n - 636 = 0
⇒ $4n^2$ + 53n - 48n - 636 = 0
⇒ n(4n + 53) - 12(4n + 53) = 0
⇒ (4n + 53) (n - 12) = 0
⇒ 4n + 53 = 0 or n - 12 = 0
$\Rightarrow n = -\frac{53}{4}$ or n = 12
$n = -\frac{53}{4}$ is in admissible as n, being the number of terms, is a natural number

$\therefore$ n = 12

Hence, 12 terms of the AP must be taken.

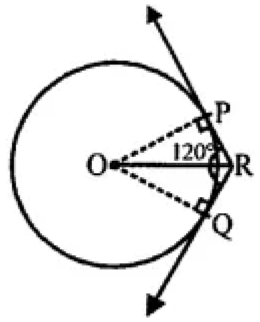
29. In the given figure, two tangents RQ and RP are drawn from the external point R to the circle with centre O.

$\angle$PRQ = 120°

To prove: OR = PR + RQ

Construction: Join OP and OQ.

Also join OR.



Proof: OR bisects the $\angle$PRQ

$\therefore \angle \mathrm{PRO} = \angle \mathrm{QRO} = \frac{120^\circ}{2} = 60^\circ$

$\because$ OP and OQ are radii and RP and RQ are tangents.

$\therefore$ OP $\perp$ PR and $\mathrm{OQ} \perp \mathrm{QR}$

In right $\triangle$ OPR

$\angle \mathrm{POR} = 180^\circ - (90^\circ + 60^\circ)$

$= 180^\circ - 150^\circ = 30^\circ$

Similarly,

$\angle \mathrm{QOR} = 30^\circ$

and $\cos\theta = \frac{\mathrm{PR}}{\mathrm{OR}}$

$\Rightarrow \cos 60^\circ = \frac{\mathrm{PR}}{\mathrm{OR}} \Rightarrow \frac{1}{2} = \frac{\mathrm{PR}}{\mathrm{OR}}$

$\Rightarrow$ 2PR=OR ......(i)

Similarly, in right $\triangle$OQR

$\Rightarrow$ 2QR=OR ....(ii)

Adding (i) and (ii)

$\Rightarrow$ 2PR + 2QR = 2OR

$\Rightarrow$ OR = PR + RQ

Hence Proved.

OR

As we know length of tangents drawn from external Pt is same

$\therefore$ BQ = BP = 2 cm

PA = AR = 5 cm

CQ = CR = 3 cm

$\therefore$ Perimeter of $\triangle$ABC

= AB + BC + CA

= AP + PB + BQ + QC + CR + AR

= 5 + 2 + 2 + 3 + 3 + 5

= 20 cm

30. We have, 4 tan θ = 3

$\Rightarrow$ tan θ = $\frac{3}{4}$

$\because sec\theta = \sqrt{1 + tan^2\theta}$ = $\sqrt{1 + \frac{9}{16}}$ = $\sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

Now, $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$

= $\frac{cos\theta(4\frac{\sin\theta}{cos\theta} - \frac{\cos\theta}{cos\theta} + \frac{1}{cos\theta})}{cos\theta(4\frac{\sin\theta}{cos\theta} + \frac{\cos\theta}{cos\theta} - \frac{1}{cos\theta})}$

= $\frac{4tan\theta - 1 + sec\theta}{4tan\theta + 1 - sec\theta}$

Substituting the values, we get,

$=\frac{4(\frac{3}{4})-1+(\frac{5}{4})}{4(\frac{3}{4})+1-(\frac{5}{4})} = \frac{3-1+(\frac{5}{4})}{3+1-(\frac{5}{4})} = \frac{2+(\frac{5}{4})}{4-(\frac{5}{4})} = \frac{\frac{8+5}{4}}{\frac{16-5}{4}} = \frac{\frac{13}{4}}{\frac{11}{4}} = \frac{13}{4} \times \frac{4}{11} = \frac{13}{11}$

31.

Weight (in kg)	**Number of students**	**Cumulative frequency**
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Now, n = 30

So, $\frac{n}{2} = \frac{30}{2} = 15$

This observation lies in the class 55-60,

So, 55-60 is the median class.

Therefore,

l = 55

h = 5

f = 6

cf = 13

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 55 + \left(\frac{15-13}{6}\right) \times 5$

$= 55 + \frac{10}{6} = 55 + \frac{5}{3}$

= 55 + 1.67 = 56.67

Hence, the median weight of the students is 56.67 kg.

Section D

32. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y-1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2

OR

Let the speed of the train be x km/hr for first 54 km and for next 63 km, speed is (x + 6) km/hr.

According to the question

$\frac{54}{x} + \frac{63}{x+6} = 3$

$\frac{54(x+6)+63x}{x(x+6)} = 3$

or, 54x + 324 + 63x = 3x(x + 6)

or, $117x + 324 = 3x^2 + 18x$

or, $3x^2 - 99x - 324 = 0$

or, $x^2 - 33x - 108 = 0$

or, $x^2 - 36x + 3x - 108 = 0$

or, x(x - 36) + 3(x - 36) = 0

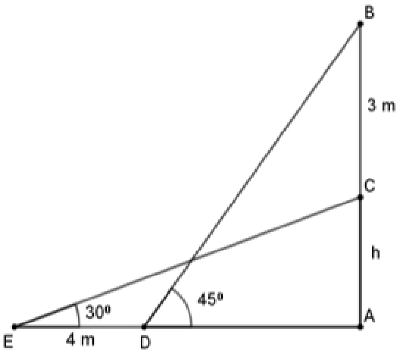
(x - 36)(x + 3) = 0

x = 36

x = - 3 rejected.

(as speed is never negative)

Hence First speed of train = 36 km/h

33. 

sin 45° = $\frac{AB}{BD}$ = $\frac{h+3}{BD}$

$\Rightarrow$ BD = h + 3$\sqrt{2}$ ...(i)

sin 30° = $\frac{1}{2}$ = $\frac{h}{CE}$

$\Rightarrow$ CE = 2h ...(ii)

length of ladder remains same

Therefore BD = CE $\Rightarrow$ (h + 3)$\sqrt{2}$ = 2h

$\Rightarrow$ h = $\frac{3\sqrt{2}}{2-\sqrt{2}}$ = $3(\sqrt{2}+1)$

Final height of the top of the ladder = $3(\sqrt{2}+1)$ m

and length of ladder = 2h = $6(\sqrt{2}+1)$ m

34. Volume of raised water in cylinder = Volume of 9000 spherical balls

$\pi(10)^2$H = $9000 \times \frac{4}{3} \times \pi \times (0.5)^3$

$\therefore$ H = 15 cm

OR

Radius of hemisphere = radius of cone = $\frac{7}{2}cm$

Height of cone = $\frac{7}{2}cm$

Volume of the solid = Volume of hemisphere + Volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(2 \times \frac{7}{2} + \frac{7}{2}\right)$

$= \frac{539}{4}$ cm^3 or 134.75 cm^3

35.

No. of wickets:	20 - 60	60 - 100	100 - 140	140 - 180	180 - 220	220 - 260	Sum
(f_i) No. of bowlers:	7	5	16	12	2	3	45
x_i	40	80	120	160	200	240	
u_i	-2	-1	0	1	2	3	
f_ix_i	-14	-5	0	12	4	9	6
cf	7	12	28	40	42	45	

Mean = $a + \frac{\sum f_i u_i}{\sum f_i} \times h = 120 + \frac{6 \times 40}{45}$ = 125.33

Median = $l + \frac{\frac{N}{2} - c}{f} \times h = 100 + \frac{22.5 - 12}{16} \times 40$ = 126.25

Section E

36. i. x + y = 300 ...(i)

150 x + 250 y = 55000 ...(ii)

ii. a. Solving equation (i) and (ii)

Number of children visited park (x) = 200

OR

b. Solving equation (i) and (ii)

Number of adults visited park (y) = 100

iii. Amount collected = 250 $\times$ 150 + 100 $\times$ 250 = ₹ 62500

37. i. $\therefore$ AF = h (Given)

$\therefore$ AF = AH + HF

h = AH + $\frac{h}{4}$

AH = h - $\frac{h}{4}$

AH = $\frac{3h}{4}$

ii. $\because$ AF = h (Given)

$\therefore$ AG = $\frac{2}{3}$ AF

$\because$ centroid divide the median in 2 : 1

iii. AH = $\frac{3h}{4}$

J is centroid of $\triangle$ADE

AJ : JH = 2 : 1

let AJ = 2x and JH = x

2x + x = $\frac{3h}{4}$

x = $\frac{h}{4}$

AJ = 2 $\times$ $\frac{h}{4}$ = $\frac{h}{2}$

AG = AJ + GJ

$= \frac{h}{2} + \frac{h}{6}$

$= \frac{2h}{3}$

But AJ = $\frac{h}{2}$ $\times$ $\frac{2}{3}$

AJ = $\frac{3}{4}$ AG

OR

GJ = AG - AJ

= AG - $\frac{3}{4}$ AG

GJ = $\frac{1}{4}$ AG

38. i. Q(x, y) is mid-point of B(-2, 4) and C(6, 4)

$\therefore$ (x, y) = $\left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right)$ = (2, 4)

ii. Since PQRS is a rhombus, therefore, PQ = QR = RS = PS.

$\therefore$ PQ = $\sqrt{(-2-2)^2 + (1-4)^2}$ = $\sqrt{16+9} = \sqrt{25}$ = 5 units

Thus, length of each side of PQRS is 5 units.

iii. Length of route PQRS = 4 PQ

= 4 $\times$ 5 = 20 units

OR

Length of CD = 4 + 2 = 6 units and length of AD = 6 + 2 = 8 units

$\therefore$ Length of route ABCD - 2(6 + 8) = 28 units